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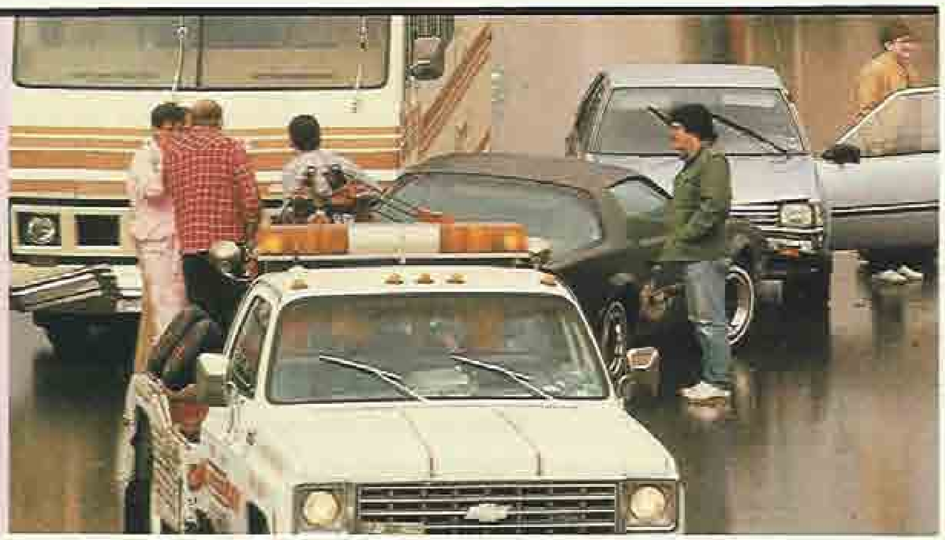
CHAPTER

5 Exponents, Roots, and Radicals

The formula for approximating the velocity V in miles per hour of a car based on the length of its skid marks S (in feet) on wet pavement is given by

$$V = 2\sqrt{3S}$$

If the skid marks are 147 feet long, what was the velocity of the car?



5–1 ■ Roots and rational exponents

The nth root

In chapter 3, we were concerned with raising some real number to a power. For example,

if $a = -4$, then $a^3 = (-4)^3 = (-4)(-4)(-4) = -64$ (read "-4 raised to the third power equals -64");
if $a = 2$, then $a^4 = (2)^4 = (2)(2)(2)(2) = 16$ (read "2 raised to the fourth power equals 16").

In this chapter, we will reverse that process. That is, we will start with a power of a real number a and find a.

Definition of nth root

For every pair of real numbers a and b and every positive integer n greater than 1, if

$$a^n = b$$

then a is called an nth root of b.

Concept

An nth root of a number is one of n equal factors that, when multiplied, equal the number.

■ Example 5–1 A

1. 2 is a fourth root of 16 since $2^4 = 16$.
2. -4 is a third root of -64 since $(-4)^3 = -64$.
3. 3 is a square root of 9 since $3^2 = 9$.
4. -3 is also a square root of 9 since $(-3)^2 = 9$. ■

We observe from these examples that both 3 and -3 are second roots (square roots) of 9 since $3^2 = 9$ and $(-3)^2 = 9$. To avoid the ambiguity of two different values for the same symbol, we now define the **principal *n*th root** of a number.

Definition of principal nth root

If n is a positive integer greater than 1 and the nth root is a real number, then the principal nth root of a nonzero real number b, denoted by $\sqrt[n]{b}$, has the same sign as the number itself. Also, the principal nth root of 0 is 0.

Since we most often talk about the principal nth root, we usually will eliminate the word "principal" and just say the "nth root," with the understanding that we are referring to the principal nth root.

Note In the expression $\sqrt[n]{b}$, the $\sqrt{}$ is called the radical symbol,* b is called the radicand, n is called the index, and $\sqrt[n]{b}$ is called the radical.

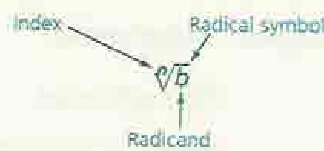


Table 5–1 lists the most common principal roots that we use in this book.

■ Table 5–1

Square roots[a]		Cube roots		Fourth roots
$\sqrt{1} = 1$	$\sqrt{64} = 8$	$\sqrt[3]{1} = 1$	$\sqrt[3]{-1} = -1$	$\sqrt[4]{1} = 1$
$\sqrt{4} = 2$	$\sqrt{81} = 9$	$\sqrt[3]{8} = 2$	$\sqrt[3]{-8} = -2$	$\sqrt[4]{16} = 2$
$\sqrt{9} = 3$	$\sqrt{100} = 10$	$\sqrt[3]{27} = 3$	$\sqrt[3]{-27} = -3$	$\sqrt[4]{81} = 3$
$\sqrt{16} = 4$	$\sqrt{121} = 11$	$\sqrt[3]{64} = 4$	$\sqrt[3]{-64} = -4$	$\sqrt[4]{256} = 4$
$\sqrt{25} = 5$	$\sqrt{144} = 12$	$\sqrt[3]{125} = 5$	$\sqrt[3]{-125} = -5$	$\sqrt[4]{625} = 5$
$\sqrt{36} = 6$	$\sqrt{169} = 13$	$\sqrt[3]{216} = 6$	$\sqrt[3]{-216} = -6$	
$\sqrt{49} = 7$	$\sqrt{196} = 14$			

[a]When we write a square root, $\sqrt{}$, the index is understood to be 2.

*The $\sqrt{}$ is the radical symbol and the $\overline{}$ means that what is under the radical symbol is a grouping.

Note Whenever we want the negative square root of a number, we indicate this by placing a minus sign in front of the square root symbol. That is, $-\sqrt{4} = -2$ and $-\sqrt{49} = -7$. Likewise, if we wanted the negative fourth root of 81, we would indicate it as $-\sqrt[4]{81} = -3$, or indicate the negative eighth root of 256 as $-\sqrt[8]{256} = -2$, and so forth.

In our definition of the principal nth root, we made the requirement that the nth root is a real number. This is because not all real numbers have a real nth root. Consider the example

$$\sqrt{-4} = \text{what?}$$

We know that all real numbers are positive, negative, or zero, and if we raise a real number to an even power (use it as a factor an even number of times), our answer will never be negative. Therefore the square root of a negative number does not exist in the set of real numbers, and, in general, **an even root of a negative number does not exist in the set of real numbers.**

Summary of nth roots

The symbol $\sqrt[n]{b}$ always represents the principal nth root of b.

	n is even	**n is odd**
If b is positive $b > 0$	Two real nth roots Principal nth root is positive	One real nth root Principal nth root is positive
If b is negative $b < 0$	NO REAL nth ROOTS	One real nth root Principal nth root is negative
If b is zero $b = 0$	One real nth root Principal nth root is zero	One real nth root Principal nth root is zero

In chapter 1, we defined the set of irrational numbers to be those numbers that cannot be represented by a terminating decimal or a nonterminating repeating decimal. Thus the nth root of b, $\sqrt[n]{b}$, is irrational if it cannot be expressed by a terminating decimal or a nonterminating repeating decimal. Some examples of irrational numbers are

$$\sqrt{8},\quad \sqrt[5]{12},\quad \sqrt[3]{-4},\quad -\sqrt{10},\quad \sqrt[4]{17}$$

Whenever we are working with irrational numbers in a problem, we may have to approximate the number to as many decimal places as are required in the problem by using a calculator or an appropriate table.

Example 5–1 B

Find the decimal approximation to three decimal places by using a calculator.

1. $\sqrt{2} \approx 1.414$ **2.** $\sqrt{15} \approx 3.873$ **3.** $-\sqrt{11} \approx -3.317$

Note "$\approx$" is read "is approximately equal to" and is used when our answer is not exact.

Quick check Find the decimal approximation to three decimal places by using a calculator. $\sqrt{17}$

Rational exponents

In chapter 3, we developed a set of properties that guided our use of integers as exponents. We will now define rational exponents so that those same properties that apply to integer exponents will also apply to rational exponents as well.

Consider the equation

$$b^{1/3} = a$$

If we raise both members of the equation to the third power, we have

$$(b^{1/3})^3 = a^3$$

Since the left member is a power to a power, we multiply the exponents to get

$$\begin{aligned} b^{1/3 \cdot 3} &= a^3 \\ b^1 &= a^3 \\ b &= a^3 \end{aligned}$$

Since $a^3 = b$, then by the definition of the principal nth root, $a = \sqrt[3]{b}$. Since we originally stated that $a = b^{1/3}$, then $b^{1/3}$ must be the same as $\sqrt[3]{b}$. That is,

$$b^{1/3} = \sqrt[3]{b}$$

Definition of $a^{1/n}$

For every real number a and positive integer n greater than 1,

$$a^{1/n} = \sqrt[n]{a}$$

whenever the principal nth root of a is a real number.

Concept

The expression $a^{1/n}$ is equivalent to $\sqrt[n]{a}$ and represents the principal nth root of a.

Example 5–1 C

Rewrite the following in radical notation. Use a calculator or table 5–1 to simplify where possible. Round the answer to three decimal places when the value is irrational.

1. $6^{1/3} = \sqrt[3]{6} \approx 1.817$

2. $81^{1/4} = \sqrt[4]{81} = 3$

3. $19^{1/2} = \sqrt{19} \approx 4.359$

4. $b^{1/7} = \sqrt[7]{b}$

Next, we need to decide how to define the expression $a^{m/n}$, where m and n are positive integers and a is a real number such that the nth root of a is also a real number. Before we determine the meaning for the expression $a^{m/n}$, we shall place a further restriction on the fraction $\frac{m}{n}$ such that $\frac{m}{n}$ is reduced to lowest terms. When the fraction $\frac{m}{n}$ is reduced to lowest terms, we say that m and n are **relatively prime.** That is, m and n contain no common positive integer factors other than 1. We can write the fraction $\frac{m}{n}$ as

$$\frac{m}{n} = \frac{1}{n} \cdot m$$

Hence we have the following definition of $a^{m/n}$.

Definition of $a^{m/n}$

For every real number a and relatively prime positive integers m and n, if the principal nth root of a is a real number, then

$$a^{m/n} = (a^{1/n})^m$$

or, equivalently,

$$a^{m/n} = (\sqrt[n]{a})^m$$

In our previous definition, the fractional exponent $\frac{m}{n}$ was rewritten as

$$\frac{1}{n} \cdot m$$

If we apply the commutative property of multiplication, we can write $\frac{1}{n} \cdot m$ as

$$m \cdot \frac{1}{n}$$

This fact leads us to the property of $(a^{1/n})^m$.

Property of $(a^{1/n})^m$

For every real number a and relatively prime positive integers m and n, if the principal nth root of a is a real number, then

$$(a^{1/n})^m = (a^m)^{1/n}$$

or, equivalently,

$$(\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

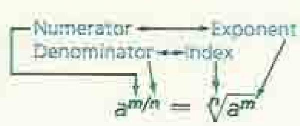
Concept

If we are dealing with a rational exponent that is reduced to lowest terms and the nth root of a is a real number, then raising the nth root of a to the mth power is equivalent to finding the nth root of a^m.

Note When we have a rational exponent such as

$$\frac{m}{n}$$

the numerator (m) indicates the power to which the base is to be raised and the denominator (n) indicates the root to be taken.



■ Example 5–1 D

Rewrite the following in radical form and use table 5–1 to simplify where possible. Assume that all variables represent positive real numbers.

1. $x^{3/4} = (\sqrt[4]{x})^3$ or $\sqrt[4]{x^3}$ Denominator becomes index, numerator becomes exponent
2. $27^{2/3} = (\sqrt[3]{27})^2 = (3)^2 = 9$
   Alternate: $27^{2/3} = \sqrt[3]{27^2} = \sqrt[3]{729} = 9$

3. $(-8)^{2/3} = (\sqrt[3]{-8})^2 = (-2)^2 = 4$
4. $-8^{2/3} = -(\sqrt[3]{8})^2 = -(2)^2 = -4$

▶ ***Quick check*** Simplify $(-27)^{2/3}$.

Example 5–1 E

Rewrite the following with rational exponents. Assume that all variables represent positive real numbers.

1. $\sqrt[3]{a^2} = a^{2/3}$ The exponent becomes the numerator, the index becomes the denominator
2. $\sqrt[5]{x} = x^{1/5}$ The power of x is understood to be 1
3. $\sqrt{5} = 5^{1/2}$ The index of the radical is understood to be 2

▶ ***Quick check*** Rewrite with rational exponents $\sqrt[7]{a}$

In the previous definition of rational exponents, we required that the values of m and n be relatively prime. The following example illustrates what happens when m and n have a common factor of 2 and the radicand is negative. Consider the expression

$$[(-4)^2]^{1/4}$$

Raising -4 to the second power, we have

$$[(-4)^2]^{1/4} = (16)^{1/4}$$

and using table 5–1 to determine the fourth root, we have

$$(16)^{1/4} = \sqrt[4]{16} = 2$$

If we take the original expression and write it as

$$[(-4)^2]^{1/4} = (-4)^{2/4} = (-4)^{1/2} = \sqrt{-4}$$

we see that the result, $\sqrt{-4}$, has no real answer. Therefore, depending on the method that we use, two different results are possible. For this reason, we have the following definition of $(a^m)^{1/n}$, when $a < 0$.

Definition of $(a^m)^{1/n}$, $a < 0$

If $a < 0$, and m and n are positive even integers,

$$(a^m)^{1/n} = |a|^{m/n}$$

Note In our example,

$$[(-4)^2]^{1/4} = |-4|^{2/4} = 4^{1/2} = \sqrt{4} = 2$$

If m and n are equal ($m = n$) and are even, the definition becomes

$$(a^n)^{1/n} = |a|^{n/n} = |a|^1 = |a|$$

or, equivalently,

$$(a^n)^{1/n} = \sqrt[n]{a^n} = |a|$$

when n is even. If $n = 2$, then

$$(a^2)^{1/2} = \sqrt{a^2} = |a|$$

In general, we make the following definition for $\sqrt[n]{a^n}$.

Definition of $\sqrt[n]{a^n}$

For every real number a and positive integer n, where $n > 1$,

$$(a^n)^{1/n} = \sqrt[n]{a^n} = \begin{cases} |a| \text{ if } n \text{ is even} \\ a \text{ if } n \text{ is odd} \end{cases}$$

Example 5–1 F

Simplify the following. Variables represent all real numbers.

1. $(x^2)^{1/2} = \sqrt{x^2} = |x|$ — Even index, absolute value is necessary
2. $b^{5/5} = \sqrt[5]{b^5} = b$ — Odd index, do not need absolute value
3. $\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$ — Odd index, do not need absolute value
4. $[(-2)^2]^{1/2} = \sqrt{(-2)^2} = |-2| = 2$ — Even index, absolute value of -2 is 2
5. $\sqrt{a^2 + 2ab + b^2} = \sqrt{(a + b)^2}$ — Factor the perfect square trinomial
   $= |a + b|$ — Even index, absolute value is necessary

▶ ***Quick check*** Simplify $(a^2)^{1/2}$ and $\sqrt{x^2 + 2xy + y^2}$. Variables represent all real numbers. ■

We now make the following definition of $a^{-m/n}$.

Definition of $a^{-m/n}$

For every real number a, $a \neq 0$, and positive integers m and n, if the principal nth root of a is a real number then,

$$a^{-m/n} = \frac{1}{a^{m/n}}$$

Example 5–1 G

Rewrite the following using positive exponents and use table 5–1 to simplify where possible.

1. $16^{-3/4} = \dfrac{1}{16^{3/4}} = \dfrac{1}{(\sqrt[4]{16})^3} = \dfrac{1}{(2)^3} = \dfrac{1}{8}$ — Rewrite the expression with a positive exponent, rewrite the expression in radical form, use table 5–1 to simplify

2. $(-8)^{-2/3} = \dfrac{1}{(-8)^{2/3}} = \dfrac{1}{(\sqrt[3]{-8})^2} = \dfrac{1}{(-2)^2} = \dfrac{1}{4}$ — Rewrite the expression with a positive exponent, rewrite the expression in radical form, use table 5–1 to simplify

Note It is only the sign of the exponent that changes, not the sign of the base. ■

Mastery points

Can you

- Find the decimal approximation for a root that is an irrational number?
- Find the principal *n*th root of a number?
- Express rational exponents in radical form?
- Express radicals in rational exponent form?

Exercise 5–1

Find the decimal approximation to three decimal places by using a calculator. See example 5–1 B.

Example $\sqrt{17}$

Solution ≈ 4.123 Fourth decimal place is 4 or less, round down

1. $\sqrt{18}$
2. $\sqrt{19}$
3. $-\sqrt{33}$
4. $-\sqrt{14}$

Rewrite the following in radical notation and use table 5–1 to simplify wherever possible. Assume that all variables represent positive real numbers. See examples 5–1 C, D, and G.

Example $(-27)^{2/3}$

Solution
$= (\sqrt[3]{-27})^2$ Rewrite in radical form
$= (-3)^2$ Simplify the radical, table 5–1
$= 9$ Standard form

5. $9^{1/3}$
6. $2^{1/2}$
7. $x^{1/2}$
8. $a^{1/3}$
9. $b^{4/5}$
10. $a^{3/4}$
11. $64^{1/3}$
12. $(-8)^{1/3}$
$\boxed{13.}$ $(-64)^{2/3}$
14. $16^{3/4}$
15. $81^{3/4}$
16. $64^{2/3}$
17. $16^{3/2}$
$\boxed{18.}$ $(-27)^{-1/3}$
19. $8^{-1/3}$
20. $49^{-1/2}$
21. $16^{-1/4}$
22. $16^{-3/4}$
23. $27^{-2/3}$
24. $(-8)^{-2/3}$
25. $(-32)^{-3/5}$
26. $(-27)^{-2/3}$
27. $x^{-3/4}$
28. $a^{-2/3}$

Rewrite the following with rational exponents. See example 5–1 E.

Example $\sqrt[7]{a}$

Solution $= a^{1/7}$ The power of *a* is understood to be 1; this is the numerator of the rational exponent; the index, 7, is the denominator

29. $\sqrt[7]{a^4}$
30. $\sqrt[9]{b}$
31. $\sqrt[5]{x}$

Simplify the following. Variables represent *all* real numbers. See example 5–1 F.

Examples $(a^2)^{1/2}$

Solutions
$= \sqrt{a^2}$ Index is even; absolute value is necessary
$= |a|$

$\sqrt{x^2 + 2xy + y^2}$
$= \sqrt{(x + y)^2}$ Factor the perfect square trinomial
$= |x + y|$ Index is even, absolute value is necessary

32. $(x^7)^{1/7}$
33. $(-8)^{3/3}$
34. $[(-3)^2]^{1/2}$
35. $[(-4)^2]^{1/2}$
36. $\sqrt{a^2 - 2ab + b^2}$
37. $\sqrt{4x^2 - 4xy + y^2}$
38. $\sqrt{a^2 + 2a + 1}$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Solve the following word problems.

39. Find the number whose principal fourth root is 4.

40. Find the number whose principal cube root is -3.

41. The electric-field intensity on the axis of a uniform charged ring is given by

$$E = \frac{T}{(x^2 + r^2)^{3/2}}$$

where T is the total charge on the ring and r is the radius of the ring. Express the rational exponent in radical form.

42. To find the velocity of the center of mass of a rolling cylinder, we use the equation

$$v = \left(\frac{4}{3}gh\right)^{1/2}$$

Write the expression in radical form.

43. The formula for approximating the velocity V in miles per hour of a car based on the length of its skid marks S (in feet) on dry pavement is given by

$$V = 2\sqrt{6S}$$

If the skid marks are 24 feet long, what was the velocity of the car?

44. When a gas is compressed with no gain or loss of heat, the pressure and the volume of the gas are related by the formula

$$p = kv^{-7/5}$$

where p represents the pressure, v represents the volume, and k is a constant. Express the formula in radical form.

Review exercises

Perform the indicated operations. Assume that all variables represent nonzero real numbers. Write each answer with only positive exponents. See sections 3–1 and 3–3.

1. $a^2 \cdot a^4 \cdot a$ **2.** $(x^2)^5$ **3.** $(2a^3b^4)^2$ **4.** $a^3 \div a^7$

5. $3x^0$ **6.** -3^2 **7.** $a^{-3} \cdot a^{-4}$ **8.** $x^{-8} \div x^{-6}$

5–2 ■ Operations with rational exponents

We have now developed the concept of rational exponents so that the same properties that applied to integer exponents can now be extended to rational exponents. The following is a restatement of those properties and definitions.

$$a^n = \overbrace{a \cdot a \cdot a \cdots a}^{n \text{ factors of } a}$$

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^nb^n$$

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}, a \neq 0$$

$$a^{-n} = \frac{1}{a^n}, a \neq 0$$

$$a^0 = 1, a \neq 0$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$$

Example 5–2 A

Perform the indicated operations and simplify. Assume that all variables represent positive real numbers. Leave the answer with only positive exponents.

1. $5^{1/3} \cdot 5^{1/3} = 5^{\frac{1}{3}+\frac{1}{3}}$ Multiplication of like bases

$= 5^{\frac{2}{3}}$ Add exponents

2. $3^{1/2} \cdot 3^{1/3} = 3^{\frac{1}{2}+\frac{1}{3}}$ Multiplication of like bases

$= 3^{\frac{3}{6}+\frac{2}{6}}$ Least common denominator is 6

$= 3^{\frac{5}{6}}$ Add exponents

3. $\dfrac{6^{1/2}}{6^{1/4}} = 6^{\frac{1}{2}-\frac{1}{4}}$ Division of like bases

$= 6^{\frac{2}{4}-\frac{1}{4}}$ Least common denominator is 4

$= 6^{\frac{1}{4}}$ Subtract exponents

4. $a^{2/3} \cdot a^{3/4} = a^{\frac{2}{3}+\frac{3}{4}}$ Multiplication of like bases

$= a^{\frac{8}{12}+\frac{9}{12}}$ Least common denominator is 12

$= a^{\frac{17}{12}}$ Add exponents

5. $(x^{4/3})^{1/2} = x^{\frac{4}{3}\cdot\frac{1}{2}}$ Power of a power

$= x^{\frac{2}{3}}$ Multiply exponents

6. $\dfrac{y^{1/3}}{y^{1/4}} = y^{\frac{1}{3}-\frac{1}{4}}$ Division of like bases

$= y^{\frac{4}{12}-\frac{3}{12}}$ Least common denominator is 12

$= y^{\frac{1}{12}}$ Subtract exponents

7. $(2^3a^{15}b^{21})^{1/3} = (2^3)^{\frac{1}{3}}(a^{15})^{\frac{1}{3}}(b^{21})^{\frac{1}{3}}$ Group of factors to a power

$= 2^{3\cdot\frac{1}{3}}a^{15\cdot\frac{1}{3}}b^{21\cdot\frac{1}{3}}$ Power of a power

$= 2^1a^5b^7$ Multiply exponents

$= 2a^5b^7$ Standard form

8. $\dfrac{x^{-1/4}y^{2/5}}{x^{3/4}y^{-4/5}} = x^{\frac{-1}{4}-\frac{3}{4}}y^{\frac{2}{5}-\frac{-4}{5}}$ Division of like bases

$= x^{\frac{-4}{4}}y^{\frac{6}{5}}$ Subtract exponents

$= x^{-1}y^{\frac{6}{5}}$ Simplify the rational exponent

$= \dfrac{y^{\frac{6}{5}}}{x^1}$ Rewrite with positive exponents

$= \dfrac{y^{\frac{6}{5}}}{x}$ Standard form

▶ ***Quick check*** Simplify $7^{\frac{1}{5}} \cdot 7^{\frac{1}{5}}$ and $(a^{\frac{5}{8}})^{\frac{2}{3}}$. ■

Mastery points

Can you

■ Apply the properties of exponents to rational exponents?

Exercise 5–2

Perform the indicated operations and simplify. Assume that all variables represent positive real numbers. Leave the answer with all exponents positive. See example 5–2 A.

Examples $7^{1/5} \cdot 7^{1/5}$ $(a^{5/8})^{2/3}$

Solutions $= 7^{1/5+1/5}$ Multiplication of like bases $= a^{5/8 \cdot 2/3}$ Power of a power
$= 7^{2/5}$ Add exponents $= a^{5/12}$ Multiply exponents

1. $2^{1/2} \cdot 2^{1/2}$
2. $a^{1/3} \cdot a^{2/3}$
3. $b^{3/4} \cdot b^{2/3}$
4. $x^{1/2} \cdot x^{5/4}$
5. $5^{3/2} \cdot 5^{-1/2}$
6. $x^{3/4} \cdot x^{-1/4}$
7. $a^{1/3} \cdot a^{-1/4}$
8. $y^{1/2} \cdot y^{-3/4}$
9. $(a^{2/3})^{4/5}$
10. $(b^{2/3})^{1/2}$
11. $(x^{3/4})^4$
12. $(a^{1/2})^{1/2}$
13. $(x^{-1/4})^4$
14. $(a^{-3/4})^{-1/3}$
15. $(b^{-2/3})^{-1/2}$
16. $(a^{1/2})^{-2/3}$
17. $(x^{-1/3})^{-3/4}$
18. $(x^{-2/3})^{-3/2}$
19. $(16y^4)^{3/4}$
20. $(a^3b^6)^{1/3}$
21. $(a^3b)^{2/3}$
22. $(8a^6b^{12})^{2/3}$
23. $(16a^{10}b^2)^{3/4}$
24. $(4a^2b^4)^{-1/2}$
25. $(27a^{12}b^3)^{-1/3}$
26. $(9x^{-2}y^4)^{-3/2}$
27. $\dfrac{y^{1/4}}{y^{1/3}}$
28. $\dfrac{a^{1/3}}{a^{5/6}}$
29. $\dfrac{b^{3/4}}{b}$
30. $\dfrac{x^{1/3}}{x}$
31. $\dfrac{x^{3/2}}{x^{-1/2}}$
32. $\dfrac{y^{-2/3}}{y^{1/3}}$
33. $\dfrac{a^{-2/3}}{a^{-4/3}}$
34. $\dfrac{x^{-1/4}}{x^{-1/3}}$
35. $\dfrac{a^{3/4}b^{1/2}}{a^{1/4}b^{1/4}}$
36. $\dfrac{xy^{3/4}}{x^{1/2}y^{1/4}}$
37. $\dfrac{ab}{a^{1/2}b^{1/2}}$
38. $\dfrac{x^{-1/2}x}{x^{1/3}}$
39. $\dfrac{b^2b^{1/3}}{b^{1/2}}$
40. $\dfrac{c^{2/3}c^{3/4}}{c^{-1/3}}$
41. $\dfrac{a^{-2/3}b^{1/2}}{a^{-1/3}b^{3/4}}$
42. $\dfrac{x^{-1/2}y^{5/4}}{x^{-2/3}y^{3/4}}$

Solve the following word problems.

43. A square-shaped television picture tube has an area of 169 square inches. What is the length of the side of the tube? (*Hint:* Area of a square is found by squaring the length of a side, $A = s^2$.)

44. A garden in the shape of a square is 196 square feet. What is the length of a side? (See exercise 43.)

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45. The formula for approximating the velocity V in miles per hour of a car based on the length of its skid marks S (in feet) on wet pavement is given by

$$V = 2\sqrt{3S}$$

If the skid marks are 75 feet long, what was the velocity of the car?

46. A tank whose shape is a cube holds 216 cubic meters of water. What is the length of an edge of the cube? (*Hint:* The volume of a cube is found by raising the length of an edge to the third power, $V = e^3$.)

47. At an altitude of h feet above the sea or level ground, the distance d in miles that a person can see an object is found by using the equation

$$d = \sqrt{\frac{3h}{2}}$$

How far can someone see who is in a tower 216 feet above the ground?

48. How can you find the fourth root of a number on a calculator using only the square root key?

49. How can you find the eighth root of a number on a calculator using only the square root key?

Review exercises

Perform the indicated multiplication. See section 3–1.

1. $(8a^3)(4a^4)$
2. $(4x^2y^2)(4x^2y)$
3. $(5x^2y^2)(75xy^2)$
4. $(3ab^2)(18a^2b^3)$
5. $(25a^5b^4)(15ab^4)$
6. $(8ab)(4a^5b^5)$

5–3 ■ Simplifying radicals—I

Product property for radicals

In this section, we will develop some properties for simplifying radicals. We will consider several forms of simplification that involve radicals. The first type of simplified radical is as follows:

The radicand (the quantity under the radical symbol) contains no factors that can be written to a power greater than or equal to the index.

The following property, called the **product property for radicals,** is useful for this type of simplification.

Product property for radicals

For all nonnegative real numbers a and b and positive integer n greater than 1,

$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$$

Concept

The nth root of a product is equal to the product of the nth roots of the factors.

To utilize this property in simplifying radicals, we look for factors that are perfect nth roots, that is, factors that are raised to the nth power. We have a perfect nth root when the value of a radical expression is a rational number. The following are examples of perfect nth roots:

$$\sqrt{25} = \sqrt{5^2} = 5 \qquad \sqrt[3]{64} = \sqrt[3]{4^3} = 4$$

$$\sqrt[4]{81} = \sqrt[4]{3^4} = 3 \qquad \sqrt[5]{32} = \sqrt[5]{2^5} = 2$$

Simplifying the principal nth root

1. If the radicand is an nth power, write the corresponding nth root.
2. If possible, factor the radicand so that at least one factor is an nth power. Write the corresponding nth root as a coefficient of the radical.
3. The nth root is in simplest form when the radicand has no nth power factors other than 1.

■ **Example 5–3 A**

Simplify the following. Assume that all variables represent nonnegative real numbers.

1. $\sqrt{18}$

Since this is a square root, we are looking for factors that are raised to the second power. Since 9 is a factor of 18 and 9 is 3^2, we have

$$\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$$

Hence, $3\sqrt{2}$ is the simplified form of $\sqrt{18}$.

2. $\sqrt[3]{32}$

The index is 3, therefore we are looking for factors raised to the third power. Since 8 is a factor of 32 and $8 = 2^3$, our problem becomes

$$\sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = \sqrt[3]{8}\sqrt[3]{4} = 2\sqrt[3]{4}$$

Hence, $2\sqrt[3]{4}$ is the simplified form of $\sqrt[3]{32}$.

3. $\sqrt[3]{a^5}$

We are looking for factors raised to the third power, and a^5 can be written as $a^3 \cdot a^2$. Hence

$$\sqrt[3]{a^5} = \sqrt[3]{a^3a^2} = \sqrt[3]{a^3}\sqrt[3]{a^2} = a\sqrt[3]{a^2}$$

4. $\sqrt[5]{a^{10}} = \sqrt[5]{a^5 \cdot a^5} = \sqrt[5]{a^5}\sqrt[5]{a^5} = a \cdot a = a^2$

Note In example 4, the exponent, 10, was a multiple of the index, 5, and the radical was eliminated. When the exponent of a factor is a multiple of the index, that factor will no longer remain under the radical symbol.

▶ ***Quick check*** Simplify $\sqrt[3]{16}$. ■

The symmetric property from chapter 1 allows us to restate the product property for radicals as follows:

$$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$$

This means that the product of n^{th} roots can be written as the n^{th} root of the product.

Example 5–3 B

Multiply the following radicals and simplify where possible. Assume that all variables represent nonnegative real numbers.

1. $\sqrt{2}\sqrt{10}$

   Since this is a square root times a square root, we can multiply the radicals together.

   $\sqrt{2}\sqrt{10} = \sqrt{2 \cdot 10} = \sqrt{20}$

   But $\sqrt{20}$ can be simplified.

   $\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

   Therefore the simplified form of $\sqrt{2}\sqrt{10}$ is $2\sqrt{5}$.

2. $\sqrt[3]{9a^2b}\sqrt[3]{9ab}$ — Indices are the same

   $= \sqrt[3]{9a^2b \cdot 9ab}$ — Multiply radicands

   $= \sqrt[3]{81a^3b^2}$ — Simplify

   $= \sqrt[3]{27 \cdot 3a^3b^2} = \sqrt[3]{27}\sqrt[3]{3}\sqrt[3]{a^3}\sqrt[3]{b^2}$

   $= 3\sqrt[3]{3} \cdot a\sqrt[3]{b^2} = 3a\sqrt[3]{3b^2}$

▶ ***Quick check*** Multiply and simplify $\sqrt{3}\sqrt{15}$

A second type of radical simplification can be seen in the following example. When we try to simplify the radical

$$\sqrt[6]{27}$$

we find that no factors of 27 can be written to a power greater than or equal to the index. Therefore it appears that no simplification is possible. But if we express the radical in rational exponent form, we observe

$$\sqrt[6]{27} = \sqrt[6]{3^3} = 3^{3/6} = 3^{1/2} = \sqrt{3}$$

We found that we could start with a radical whose index is 6 and reduce the index to 2 (square root). In reducing the index of the radical, it has been simplified. Therefore a second way that a radical is simplified is when **the exponent of the radicand and the index of the radical have no common factor other than 1.** That is, the exponent and the index are relatively prime.

Property $\sqrt[kn]{a^{km}}$

If a is a positive real number, m is an integer, and n and k are positive integers greater than 1, then

$$\sqrt[kn]{a^{km}} = \sqrt[n]{a^m}$$

Concept

We can divide out a common factor between the index and the exponent of the radicand.

■ Example 5–3 C

Simplify the following. Assume that all variables represent positive real numbers.

1. $\sqrt[6]{9} = \sqrt[6]{3^2}$

   Since the exponent of the radicand and the index of the radical both have a common factor of 2, we can divide out (cancel) the common factor.

   $\sqrt[6]{3^2} = 3^{2/6} = 3^{1/3} = \sqrt[3]{3^1} = \sqrt[3]{3}$

2. $\sqrt[9]{x^3y^6}$

   The radicand can be written as $(xy^2)^3$, and we see from this that there is a common factor of 3 in the index and exponent.

   $\sqrt[9]{x^3y^6} = \sqrt[9]{(xy^2)^3} = (xy^2)^{3/9} = (xy^2)^{1/3} = \sqrt[3]{(xy^2)^1} = \sqrt[3]{xy^2}$

Note When the radicand contains two or more different factors, we can reduce the index *only* if there is a common factor in the index and each of the exponents of the *different* factors. For example, $\sqrt[7]{a^4b^3c^6}$ is in simplest form because 1 is the only common factor between the index and the three exponents. ■

Mastery points

Can you

- Simplify radicals by using the product property for radicals?
- Multiply radicals with the same indices?
- Simplify radicals by reducing the index of the radical?

Exercise 5–3

Simplify the following. Assume that all variables represent positive real numbers. See examples 5–3 A, B, and C.

Examples $\sqrt[3]{16}$

Solutions

$= \sqrt[3]{8 \cdot 2}$ — $8 = 2^3$ and is a factor of 16
$= \sqrt[3]{8}\sqrt[3]{2}$ — Product property
$= 2\sqrt[3]{2}$ — Simplified form

$\sqrt{3}\sqrt{15}$

$= \sqrt{3 \cdot 15}$ — Indices are the same, multiply the radicands
$= \sqrt{45}$ — Simplify
$= \sqrt{9 \cdot 5}$ — $9 = 3^2$ and 45 can be written $9 \cdot 5$
$= \sqrt{9}\sqrt{5}$ — Product property
$= 3\sqrt{5}$ — Simplified form

1. $\sqrt{20}$
2. $\sqrt{63}$
3. $\sqrt[3]{24}$
4. $\sqrt[3]{32}$
5. $\sqrt[4]{a^5}$
6. $\sqrt[5]{a^7}$
7. $\sqrt[9]{a^{18}}$
8. $\sqrt[5]{b^{10}}$
9. $\sqrt[5]{c^5}$
10. $\boxed{10.}$ $\sqrt{9x^2y^5}$
11. $\sqrt{25x^3y^9}$
12. $\sqrt{32a^4b^7}$
13. $\sqrt{50a^6bc^9}$
14. $\sqrt[3]{8x^5y^4}$
15. $\sqrt[3]{27a^3b^2c^{12}}$
16. $\sqrt[3]{16a^4b^5}$
17. $\sqrt[3]{81a^5b^{11}}$
18. $\sqrt[5]{64x^{10}y^{14}}$
19. $\sqrt[5]{32a^{10}b^4c^{12}}$
20. $\sqrt[5]{8a^7b^{15}c^3}$
21. $\sqrt{x^2 + 6x + 9}$
22. $\sqrt{a^2 + 10a + 25}$
23. $\sqrt{9a^2 + 6a + 1}$
24. $\boxed{24.}$ $\sqrt{x^2 + y^2}$
25. $\sqrt{6}\sqrt{27}$
26. $\sqrt{10}\sqrt{20}$
27. $\sqrt{7}\sqrt{7}$
28. $\sqrt{12}\sqrt{12}$
29. $\sqrt{6}\sqrt{24}$
30. $\sqrt[3]{4}\sqrt[3]{4}$
31. $\sqrt[3]{6}\sqrt[3]{12}$
32. $\sqrt[3]{10}\sqrt[3]{4}$
33. $\sqrt{3a}\sqrt{15a}$
34. $\sqrt{7b}\sqrt{14b}$
35. $\sqrt[3]{a^2}\sqrt[3]{a^2}$
36. $\sqrt[5]{b^4}\sqrt[5]{b^3}$
37. $\sqrt[4]{x}\sqrt[4]{x^3}$
38. $\sqrt[3]{2x^2}\sqrt[3]{2x}$
39. $\sqrt[5]{8x^4}\sqrt[5]{4x^3}$
40. $\boxed{40.}$ $\sqrt[3]{4a^2b}\sqrt[3]{4a^2b^2}$

41. $\sqrt[3]{5a^2b}\sqrt[3]{75a^2b^2}$ **42.** $\sqrt[3]{3ab^2}\sqrt[3]{18a^2b^2}$ **43.** $\sqrt[3]{25x^5y^7}\sqrt[3]{15xy^3}$ **44.** $\sqrt[3]{16a^{11}b^4}\sqrt[3]{12a^4b^6}$

45. $\sqrt[4]{8xy}\sqrt[4]{4x^3y^3}$ **46.** $\sqrt[6]{a^3}$ **47.** $\sqrt[10]{y^5}$ $\boxed{\textbf{48.}}$ $\sqrt[6]{b^{10}}$

49. $\sqrt[8]{y^{14}}$ **50.** $\sqrt[4]{4x^2}$ **51.** $\sqrt[6]{8y^3}$ **52.** $\sqrt[9]{27x^6y^6}$

53. $\sqrt[9]{8a^3b^6}$

Simplify the following. Variables represent *all* real numbers. See example 5–1 F.

Example $\sqrt{25a^2}$

Solution $= \sqrt{25}\sqrt{a^2}$ Product property
$= 5|a|$ Index is even, absolute value is necessary

Example $\sqrt{x^2 + 2xy + y^2}$

Solution $= \sqrt{(x + y)^2}$ Factor the trinomial
$= |x + y|$ Index is even, absolute value is necessary

54. $\sqrt{9a^2}$ **55.** $\sqrt{16x^2}$ **56.** $\sqrt{36a^2b^4}$ **57.** $\sqrt{49b^2c^2}$

58. $\sqrt{x^2 + 6x + 9}$ **59.** $\sqrt{a^2 - 8a + 16}$ **60.** $\sqrt{9x^2 + 6xy + y^2}$ **61.** $\sqrt{a^2b}$

62. $\sqrt[3]{8x^3y}$ **63.** $\sqrt[3]{27ab^3}$ **64.** $\sqrt[4]{16x^3y^4}$ **65.** $\sqrt[4]{81a^4b^3}$

66. For what values of x is $\sqrt[4]{4x^2} = \sqrt{2x}$ a false statement?

Solve the following word problems.

$\boxed{\textbf{67.}}$ The moment of inertia for a rectangle is given by the formula

$$I = \frac{bh^3}{12}$$

If we know the values of I and b, we can solve for h as follows:

$$h = \sqrt[3]{\frac{12I}{b}}$$

Find h if $I = 27$ in.^{4} and $b = 4$ in.

68. Use exercise 67 to find h if $I = 2$ in.^{4} and $b = 3$ in.

69. The moment of inertia for a circle is given by the formula

$$I = \frac{\pi r^4}{4}$$

If we know the value of I, we can solve for r as follows:

$$r = \sqrt[4]{\frac{4I}{\pi}}$$

Find r if $I = 63.585$ in.^{4} and we use 3.14 for π.

70. Use exercise 69 to find r if $I = 12.56$ in.^{4}.

71. The formula for finding the length of an edge e of a cube when the volume v is known is $e = \sqrt[3]{v}$. What is the length of the edge of a cube whose volume is 216 cubic units?

72. What is the length of the edge of a cube whose volume is 729 cubic units? (Refer to exercise 71.)

73. The current I (amperes) in a circuit is found by the formula

$$I = \sqrt{\frac{\text{watts}}{\text{ohms}}}$$

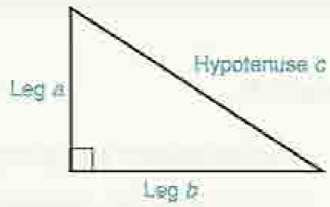
What is the current of a circuit that has 3-ohms resistance and uses 450 watts?

74. What is the current of a circuit that has 2-ohms resistance and uses 1,728 watts? (Refer to exercise 73.)

The following problems will make use of an important property of right triangles called the **Pythagorean Theorem.**

In a right triangle, the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the lengths of the legs (the sides that form the right angle). If c is the hypotenuse and a and b are the lengths of the legs, this property can be stated as:

$c^2 = a^2 + b^2$ or $c = \sqrt{a^2 + b^2}$
Also as $a^2 = c^2 - b^2$ or $a = \sqrt{c^2 - b^2}$
Also as $b^2 = c^2 - a^2$ or $b = \sqrt{c^2 - a^2}$

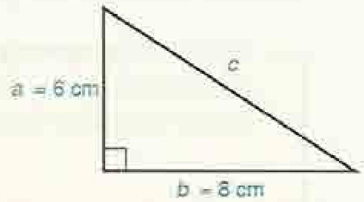


Example Find the length of the hypotenuse of a right triangle whose legs are 6 cm and 8 cm.

Solution We want to find c when $a = 6$ cm and $b = 8$ cm.

By the Pythagorean Theorem,

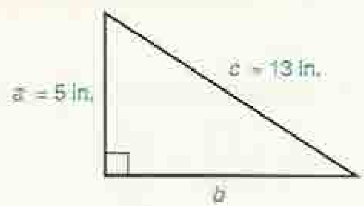
$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{6^2 + 8^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} = 10. \text{ Hence } c = 10 \text{ cm.} \end{aligned}$$



Example Find the second leg of a right triangle whose hypotenuse has length 13 in. and the first leg is 5 in. long.

Solution We want to find b given $c = 13$ in. and $a = 5$ in. Using one of the forms of the Pythagorean Theorem,

$$\begin{aligned} b &= \sqrt{c^2 - a^2} \\ &= \sqrt{13^2 - 5^2} \\ &= \sqrt{169 - 25} \\ &= \sqrt{144} = 12. \text{ Hence } b = 12 \text{ in.} \end{aligned}$$



Find the length of the unknown side in the following right triangles.

75. $a = 3$ m, $b = 4$ m
76. $a = 8$ ft, $c = 10$ ft
77. $a = 12$ in., $b = 5$ in.
78. $a = 15$ cm, $b = 8$ cm
$\boxed{79.}$ $a = 5$ in., $b = 4$ in.
80. $a = 6$ yd, $c = 10$ yd
81. $a = 6$ ft, $b = 9$ ft
82. $b = 16$ m, $c = 20$ m
83. $a = 12$ mm, $b = 16$ mm
84. $a = 10$ in., $b = 24$ in.
85. $a = 4$ cm, $c = 10$ cm

Solve the following word problems.

86. A 17-foot ladder is placed against the wall of a house. If the bottom of the ladder is 8 feet from the house, how far from the ground is the top of the ladder?

87. Find the width of a rectangle whose diagonal is 13 feet and length is 12 feet.

88. Find the diagonal of a rectangle whose length is 8 meters and whose width is 6 meters.

89. Under ideal conditions, the velocity v in meters per second of an object falling freely from a height h is given by $v = \sqrt{2gh}$, where g is the acceleration due to gravity. Use a calculator to find the velocity when h is 100 m. Round to two decimal places. Use $g = 9.8$ m/s^2.

Review exercises

Simplify the following expressions. See section 5–1.

1. $\sqrt{81}$ **2.** $\sqrt[3]{64}$ **3.** $\sqrt[5]{32}$ **4.** $\sqrt[3]{a^3}$ **5.** $\sqrt[5]{x^5}$ **6.** $\sqrt[5]{a^2 \cdot a^3}$ **7.** $\sqrt[3]{2 \cdot 4}$ **8.** $\sqrt[7]{x^5 \cdot x^2}$

5–4 ■ Simplifying radicals—II

The quotient property for radicals

In this section, we continue to develop some of the properties for simplifying radicals. A third way that a radical is simplified is when **the radicand contains no fractions.** The following property, called the **quotient property for radicals,** is useful for this type of simplification.

Quotient property for radicals

For all nonnegative real numbers a and b, where $b \neq 0$, and positive integer n greater than 1,

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Concept

The nth root of a quotient can be written as the nth root of the numerator divided by the nth root of the denominator.

■ Example 5–4 A

Simplify the following. Assume that all variables represent positive real numbers.

1. $\sqrt{\frac{5}{16}} = \frac{\sqrt{5}}{\sqrt{16}}$ No simplification can be done inside the radical, apply quotient property

$= \frac{\sqrt{5}}{4}$ Simplify radical in denominator

2. $\sqrt{\frac{a^3}{b^2}} = \frac{\sqrt{a^3}}{\sqrt{b^2}}$ No simplification can be done inside the radical, apply quotient property

$= \frac{a\sqrt{a}}{b}$ Simplify radicals

3. $\sqrt[3]{\frac{x^5y^2}{x^2}} = \sqrt[3]{x^3y^2}$ Reduce fraction by the common factor x^2

$= \sqrt[3]{x^3}\sqrt[3]{y^2}$ Product property

$= x\sqrt[3]{y^2}$ Simplify first radical ■

Rationalizing a denominator that has a single term

When the problem has been simplified and a radical still remains in the denominator, the evaluation of the problem usually is an involved process. For this reason, the fourth way that a radical is simplified is when **no radicals appear in the denominator.** This procedure is called **rationalizing the denominator,** because it changes the denominator from a radical (irrational number) to a rational number.

Consider the example

$$\sqrt{\frac{4}{5}} = \frac{\sqrt{4}}{\sqrt{5}} = \frac{2}{\sqrt{5}}$$

We would like to eliminate the radical symbol in the denominator. To remove this radical, we multiply the numerator and the denominator by a radical that yields a perfect square radicand and thereby allows us to eliminate the radical in the denominator.

$$\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{25}} = \frac{2\sqrt{5}}{5}$$

Since we multiplied the numerator and the denominator by the same number, using the fundamental principle of fractions, our answer is equivalent to the original fraction.

Example 5–4 B

Simplify the following. Assume that all variables represent positive real numbers.

1. $\sqrt{\dfrac{2}{3}} = \dfrac{\sqrt{2}}{\sqrt{3}}$ — Quotient property

$= \dfrac{\sqrt{2}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{2 \cdot 3}}{\sqrt{3 \cdot 3}}$ — Multiply by $\dfrac{\sqrt{3}}{\sqrt{3}}$ to rationalize denominator

$= \dfrac{\sqrt{6}}{\sqrt{9}}$ — We now have a perfect square root and can eliminate the radical in the denominator

$= \dfrac{\sqrt{6}}{3}$ — Expression is simplified

2. $\sqrt{\dfrac{a^3}{b}} = \dfrac{\sqrt{a^3}}{\sqrt{b}}$ — Quotient property

$= \dfrac{a\sqrt{a}}{\sqrt{b}}$ — Simplify numerator: product property

$= \dfrac{a\sqrt{a}}{\sqrt{b}} \cdot \dfrac{\sqrt{b}}{\sqrt{b}}$ — Multiply by $\dfrac{\sqrt{b}}{\sqrt{b}}$

$= \dfrac{a\sqrt{ab}}{\sqrt{b^2}}$ — Multiply radicals

$= \dfrac{a\sqrt{ab}}{b}$ — Eliminate radical in denominator

The following example will help us develop a general procedure for rationalizing a denominator that has a single term.

$$\sqrt[3]{\frac{1}{a}} = \frac{\sqrt[3]{1}}{\sqrt[3]{a}} = \frac{1}{\sqrt[3]{a}}$$

At this point, a radical remains in the denominator. We must now determine what we can do to the fraction to remove the radical from the denominator.

Observations:

1. We can multiply the numerator and the denominator by the same number and form equivalent fractions.
2. If we multiply by a radical, the indices must be the same to carry out the multiplication.

3. To bring a factor out from under the radical symbol and not leave any of the factor behind, the exponent must be a multiple of the index.

With these observations in mind, we rationalize the fraction.

$= \dfrac{1}{\sqrt[3]{a}} \cdot \dfrac{\sqrt[3]{a^2}}{\sqrt[3]{a^2}}$ The indices are the same and we multiply the numerator and the denominator by the same number

$= \dfrac{\sqrt[3]{a^2}}{\sqrt[3]{a \cdot a^2}} = \dfrac{\sqrt[3]{a^2}}{\sqrt[3]{a^3}}$ The sum of the exponents of a in the denominator adds to the index, forming a perfect nth root (cube root)

$= \dfrac{\sqrt[3]{a^2}}{a}$ The radical is eliminated from the denominator

Procedure for rationalizing a denominator of one term

1. We multiply the numerator and the denominator by a radical with the same index as the radical that we wish to eliminate from the denominator.
2. The exponent of the factor under the radical must be such that when we add it to the original exponent of the factor under the radical in the denominator, the sum will be equal to, or a multiple of, the index of the radical.
3. We carry out the multiplication and reduce the fraction if possible.

Example 5–4 C

Simplify the following. Assume that all variables represent positive real numbers.

1. $\dfrac{1}{\sqrt[5]{x^2}}$ To eliminate the radical, we multiply by another 5th root where the exponents of x will add up to 5 $(x^2 \cdot x^3 = x^{2+3} = x^5)$

$= \dfrac{1}{\sqrt[5]{x^2}} \cdot \dfrac{\sqrt[5]{x^3}}{\sqrt[5]{x^3}} = \dfrac{\sqrt[5]{x^3}}{\sqrt[5]{x^2 \cdot x^3}}$

$= \dfrac{\sqrt[5]{x^3}}{\sqrt[5]{x^5}}$ The resulting denominator is a perfect nth root (5th root) and the radical symbol is eliminated

$= \dfrac{\sqrt[5]{x^3}}{x}$

2. $\dfrac{x}{\sqrt[4]{x}} = \dfrac{x}{\sqrt[4]{x}} \cdot \dfrac{\sqrt[4]{x^3}}{\sqrt[4]{x^3}} = \dfrac{x\sqrt[4]{x^3}}{\sqrt[4]{x^4}} = \dfrac{x\sqrt[4]{x^3}}{x} = \sqrt[4]{x^3}$

3. $\dfrac{\sqrt[5]{a^3}}{\sqrt[5]{b^2}} = \dfrac{\sqrt[5]{a^3}}{\sqrt[5]{b^2}} \cdot \dfrac{\sqrt[5]{b^3}}{\sqrt[5]{b^3}} = \dfrac{\sqrt[5]{a^3b^3}}{\sqrt[5]{b^5}} = \dfrac{\sqrt[5]{a^3b^3}}{b}$

4. $\dfrac{1}{\sqrt[5]{a^2b}}$

Since there are two different factors under the radical, the radicand of the radical that we multiply by must contain a and b with exponents such that the resulting radicand in the denominator is a^5b^5. $(a^2b \cdot a^3b^4 = a^{2+3}b^{1+4} = a^5b^5)$ Therefore, we multiply the numerator and the denominator by $\sqrt[5]{a^3b^4}$.

$= \dfrac{1}{\sqrt[5]{a^2b}} \cdot \dfrac{\sqrt[5]{a^3b^4}}{\sqrt[5]{a^3b^4}} = \dfrac{\sqrt[5]{a^3b^4}}{\sqrt[5]{a^2ba^3b^4}} = \dfrac{\sqrt[5]{a^3b^4}}{\sqrt[5]{a^5b^5}}$ Eliminate the radical in the denominator

$= \dfrac{\sqrt[5]{a^3b^4}}{ab}$

▶ ***Quick check*** Assume that all variables represent positive real numbers. Simplify $\dfrac{a}{\sqrt[3]{a}}$ and $\sqrt{\dfrac{x^5}{y}}$.

The following is a summary of the conditions necessary for a radical expression to be in **simplest form,** also called **standard form.**

Standard form for a radical expression

1. The radicand contains no factors that can be written to an exponent greater than or equal to the index. ($\sqrt[3]{a^4}$ violates this.)
2. The exponent of the radicand and the index of the radical have no common factor other than 1. ($\sqrt[9]{a^6}$ violates this.)
3. The radicand contains no fractions. $\left(\sqrt{\dfrac{a}{b}}\text{ violates this.}\right)$
4. No radicals appear in the denominator. $\left(\dfrac{1}{\sqrt{a}}\text{ violates this.}\right)$

Mastery points

Can you

- Simplify radicals containing fractions by using the quotient property for radicals?
- Rationalize fractions whose denominators are a single term?

Exercise 5–4

Simplify the following expressions leaving no radicals in the denominator. Assume that all variables represent positive real numbers. See examples 5–4 A, B, and C.

Examples $\sqrt{\dfrac{x^5}{y}}$

Solutions

$= \dfrac{\sqrt{x^5}}{\sqrt{y}}$ Quotient property

$= \dfrac{x^2\sqrt{x}}{\sqrt{y}}$ Product property

$= \dfrac{x^2\sqrt{x}}{\sqrt{y}} \cdot \dfrac{\sqrt{y}}{\sqrt{y}}$ Multiply by $\dfrac{\sqrt{y}}{\sqrt{y}}$

$= \dfrac{x^2\sqrt{xy}}{\sqrt{y^2}}$ Product property

$= \dfrac{x^2\sqrt{xy}}{y}$ Radical symbol is eliminated from the denominator

$\dfrac{a}{\sqrt[3]{a}}$

$= \dfrac{a}{\sqrt[3]{a}} \cdot \dfrac{\sqrt[3]{a^2}}{\sqrt[3]{a^2}}$ Multiply by $\dfrac{\sqrt[3]{a^2}}{\sqrt[3]{a^2}}$ to rationalize

$= \dfrac{a\sqrt[3]{a^2}}{\sqrt[3]{a^3}}$

$= \dfrac{a\sqrt[3]{a^2}}{a}$ Eliminate radical symbol from denominator

$= \sqrt[3]{a^2}$ Reduce fraction

1. $\sqrt{\dfrac{16}{25}}$ **2.** $\sqrt{\dfrac{25}{49}}$ **3.** $\sqrt{\dfrac{7}{9}}$ **4.** $\sqrt{\dfrac{3}{4}}$ **5.** $\sqrt[3]{\dfrac{8}{27}}$

6. $\sqrt[3]{\dfrac{27}{64}}$ **7.** $\sqrt{\dfrac{a^6}{9}}$ **8.** $\sqrt[3]{\dfrac{4x^9}{y^6}}$ **9.** $\sqrt[3]{\dfrac{2x}{y^{15}}}$ **10.** $\sqrt[5]{\dfrac{32a^3}{b^{15}}}$

11. $\sqrt[7]{\frac{x^{21}}{y^7z^{14}}}$ **12.** $\sqrt[3]{\frac{a^7b^2}{ab^5}}$ **13.** $\sqrt[3]{\frac{16x^4}{2xy^6}}$ **14.** $\sqrt[5]{\frac{2a^{12}b^4}{64b^9}}$ **15.** $\sqrt[5]{\frac{64x^{14}y^6}{x^4y}}$

16. $\sqrt[4]{\frac{b^4c^9}{a^{11}}}$ **17.** $\sqrt{\frac{1}{2}}$ **18.** $\sqrt{\frac{1}{3}}$ **19.** $\sqrt{\frac{9}{10}}$ **20.** $\sqrt{\frac{4}{11}}$

21. $\sqrt{\frac{1}{6}}$ **22.** $\sqrt{\frac{1}{10}}$ **23.** $\sqrt{\frac{9}{50}}$ **24.** $\sqrt{\frac{7}{12}}$ **25.** $\frac{2}{\sqrt{2}}$

26. $\frac{6}{\sqrt{3}}$ **27.** $\frac{12}{\sqrt{8}}$ **28.** $\frac{18}{\sqrt{27}}$ **29.** $\sqrt[3]{\frac{27}{4}}$ **30.** $\sqrt[3]{\frac{9}{25}}$

31. $\sqrt[5]{\frac{32}{81}}$ **32.** $\sqrt[3]{\frac{27}{16}}$ **33.** $\sqrt[4]{\frac{81}{64}}$ **34.** $\sqrt[4]{\frac{2}{9}}$ **35.** $\sqrt{\frac{x^2}{y}}$

36. $\sqrt{\frac{1}{b}}$ **37.** $\sqrt{\frac{1}{c}}$ **38.** $\sqrt{\frac{x^4}{y^3}}$ **39.** $\sqrt[3]{\frac{a^3}{b^2}}$ **40.** $\sqrt[3]{\frac{b^9}{c}}$

41. $\sqrt[3]{\frac{a}{b^2}}$ **42.** $\sqrt[3]{\frac{a}{b}}$ **43.** $\sqrt[5]{\frac{32x^5}{y^2}}$ **44.** $\frac{x^2}{\sqrt[3]{x^2}}$ **45.** $\frac{ab}{\sqrt[5]{b^4}}$

46. $\sqrt{\frac{a}{bc}}$ **47.** $\sqrt{\frac{2x}{yz}}$ **48.** $\sqrt[3]{\frac{x^3}{y^2z}}$ **49.** $\sqrt[3]{\frac{8x}{y^2z}}$ **50.** $\sqrt[3]{\frac{1}{2ab^2}}$

51. $\sqrt[7]{\frac{1}{16x^2y^3}}$ **52.** $\sqrt[4]{\frac{16}{a^3b^2}}$ **53.** $\frac{x}{\sqrt[5]{x^2y^4}}$ **54.** $\frac{a}{\sqrt[4]{ab^3}}$ **55.** $\frac{xy}{\sqrt[3]{xy^2}}$

56. $\frac{ab^2}{\sqrt[5]{a^4b^{12}}}$ **57.** $\frac{b^2c}{\sqrt[7]{b^4c^3}}$ **58.** $\frac{y^2z^3}{\sqrt[5]{y^7z^2}}$ **59.** $\sqrt{\frac{2y}{x}}\sqrt{\frac{x^2}{8}}$ **60.** $\sqrt{\frac{3a}{b^3}}\sqrt{\frac{ab}{27}}$

61. $\sqrt[3]{\frac{16a^7}{b^4}}\sqrt[3]{\frac{b}{2a}}$ **62.** $\sqrt[3]{\frac{3y}{x^7}}\sqrt[3]{\frac{x}{81y^4}}$ **63.** $\sqrt[5]{\frac{x^2y}{z^7}}\sqrt[5]{\frac{y^9z^3}{x^7}}$ **64.** $\sqrt[5]{\frac{a^3b}{c^8}}\sqrt[5]{\frac{b^4c^4}{a^8}}$

Solve the following word problems. Assume that all variables represent positive real numbers.

65. If we wish to construct a sphere of specific volume, V, we can find the length of the radius, r, necessary by the formula

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

Find the radius necessary for a sphere to have a volume of 904.32 cubic units. (Use 3.14 for π.)

66. Use exercise 65 to find r if $V = 113.04$ cubic units. (Use 3.14 for π.)

67. To find the velocity of the center of mass of a rolling cylinder, we use the equation

$$v = \left(\frac{4}{3}gh\right)^{1/2}$$

Write the expression in radical form and leave the answer in standard form.

68. When a gas is compressed with no gain or loss of heat, the pressure and volume of the gas are related by the formula

$$p = kv^{-7/5}$$

where p represents pressure, v represents volume, and k is a constant. Express the formula in radical form and leave the answer in standard form.

69. The formula below gives the length s of the side of an isosceles right triangle with hypotenuse c. Express the radical in standard form.

$$s = \sqrt{\frac{c^2}{2}}$$

70. If we know the length of the diagonal d of a square, we can find the length of the side s of the square using the formula

$$s = \sqrt{\frac{d^2}{2}}$$

Express the radical in standard form.

71. The formula below gives the diagonal length d of a regular hexagon, where f is the distance across the flats. Express the radical in standard form.

$$d = \sqrt{\frac{4f^2}{3}}$$

72. The resonant frequency f of an AC series circuit is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Express the radical in standard form.

73. The average speed v of a molecule of an ideal gas is given by

$$v = \sqrt{\frac{8kT}{\pi m}}$$

where m is the mass, T is the absolute temperature, and k is the Boltzmann constant. Express the radical in standard form.

74. The formula below is used to find the potential energy in a truss. Express the radical in standard form.

$$C = \frac{3KA}{\sqrt[3]{1{,}024L}}$$

Review exercises

Perform the indicated addition or subtraction. See sections 1–6 and 4–3.

1. $5x^2 - 2x + 3x + 4x^2$ **2.** $5a^2b - ab^2 - 3a^2b$ **3.** $a^3 + 3a^2 + 4a^3 - a^2$ **4.** $x^2y - 3xy^2 + 2x^2y - xy^2$

5. $\frac{3}{4} + \frac{5}{2}$ **6.** $\frac{1}{2x} + \frac{3}{x}$ **7.** $\frac{15}{9a} - \frac{1}{a}$ **8.** $\frac{5}{3x} + \frac{2}{4x}$

5–5 ■ Sums and differences of radicals

We have learned that we can only combine like terms in addition and subtraction. This same idea applies when we are dealing with radicals. *We can only add or subtract like radicals.*

To have like radicals, the following must be true:

1. The radicals must have the same index.
2. The radicands must be the same.

For example, the expressions $-3\sqrt[5]{11}$ and $4\sqrt[5]{11}$ are like radicals since the indices, 5, are the same and the radicands, 11, are the same. The expressions $3\sqrt[5]{19}$ and $3\sqrt[4]{19}$ are not like radicals because the indices are different $(4 \neq 5)$, and the expressions $7\sqrt[3]{13}$ and $7\sqrt[3]{14}$ are not like radicals because the radicands are different $(13 \neq 14)$.

Addition and subtraction of radicals follow the same procedure as addition and subtraction of algebraic expressions.

To combine like radicals

1. Perform any simplification within the terms.
2. Use the distributive property to combine terms that have like radicals.

Example 5–5 A

Perform the indicated addition and subtraction. Assume that all variables represent nonnegative real numbers.

1. $3\sqrt{5} + 2\sqrt{5}$

   Since we have like radicals, we can perform the addition by using the distributive property.

   $3\sqrt{5} + 2\sqrt{5} = (3 + 2)\sqrt{5} = 5\sqrt{5}$

2. $8\sqrt[5]{x^2} - 2\sqrt[5]{x^2} + 3\sqrt[5]{x^2} = (8 - 2 + 3)\sqrt[5]{x^2} = 9\sqrt[5]{x^2}$

3. $4\sqrt{a} + \sqrt{b} - 2\sqrt{a} + 3\sqrt{b}$

   Using the commutative and associative properties, we group the like radicals

   $= (4\sqrt{a} - 2\sqrt{a}) + (\sqrt{b} + 3\sqrt{b})$

   and applying the distributive property, we perform the addition and subtraction.

   $= (4 - 2)\sqrt{a} + (1 + 3)\sqrt{b}$
   $= 2\sqrt{a} + 4\sqrt{b}$

   Since $\sqrt{a}$ and $\sqrt{b}$ are not like radicals, no further simplification can be performed.

Consider the problem

$$\sqrt{27} + 4\sqrt{3}$$

It appears that the indicated addition cannot be performed since we do not have like radicals. However we should have observed that the $\sqrt{27}$ can be simplified.

$$\sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$$

Our problem then becomes

$$\sqrt{27} + 4\sqrt{3} = 3\sqrt{3} + 4\sqrt{3} = (3 + 4)\sqrt{3} = 7\sqrt{3}$$

Therefore *whenever we are working with radicals, we must be certain that all radicals are in simplest form.*

Example 5–5 B

Perform the indicated addition and subtraction.

1. $5\sqrt{2} + \sqrt{18}$

   Since $\sqrt{18}$ can be simplified, we have

   $= 5\sqrt{2} + \sqrt{9 \cdot 2} = 5\sqrt{2} + \sqrt{9}\sqrt{2} = 5\sqrt{2} + 3\sqrt{2}$

   and applying the distributive property,

   $= (5 + 3)\sqrt{2} = 8\sqrt{2}$

2. $\sqrt{27} + \sqrt{12}$ — Simplify radicals

   $= \sqrt{9 \cdot 3} + \sqrt{4 \cdot 3}$ — Look for factors that are squares
   $= 3\sqrt{3} + 2\sqrt{3}$ — Add like radicals
   $= 5\sqrt{3}$

3. $4\sqrt[3]{81} - \sqrt[3]{24}$ Simplify radicals

$= 4\sqrt[3]{27 \cdot 3} - \sqrt[3]{8 \cdot 3}$ Look for factors that are cubes

$= 4 \cdot 3\sqrt[3]{3} - 2\sqrt[3]{3}$

$= 12\sqrt[3]{3} - 2\sqrt[3]{3}$ Subtract like radicals

$= 10\sqrt[3]{3}$

▶ ***Quick check*** Add $3\sqrt{3} + \sqrt{12}$.

When we perform addition and subtraction of fractions that contain radicals, our first step is to simplify all radicals involved. We can then find the least common denominator and perform the indicated addition and subtraction.

Example 5–5 C

Perform the indicated addition and subtraction.

1. $\dfrac{2}{3} + \dfrac{1}{\sqrt{3}}$ Simplify the second fraction by rationalizing the denominator.

$$= \frac{2}{3} + \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2}{3} + \frac{\sqrt{3}}{\sqrt{9}} = \frac{2}{3} + \frac{\sqrt{3}}{3}$$

Since the denominators are the same, we can add the numerators and write the sum over the common denominator.

$$\frac{2}{3} + \frac{\sqrt{3}}{3} = \frac{2 + \sqrt{3}}{3}$$

2. $\dfrac{3}{\sqrt{2}} - \dfrac{2}{\sqrt{5}} = \dfrac{3}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} - \dfrac{2}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$ Rationalize the denominators

$$= \frac{3\sqrt{2}}{\sqrt{4}} - \frac{2\sqrt{5}}{\sqrt{25}} = \frac{3\sqrt{2}}{2} - \frac{2\sqrt{5}}{5}$$

The least common denominator is 10. Therefore we multiply the first fraction by $\dfrac{5}{5}$ and the second fraction by $\dfrac{2}{2}$.

$$\frac{3\sqrt{2}}{2} \cdot \frac{5}{5} - \frac{2\sqrt{5}}{5} \cdot \frac{2}{2} = \frac{15\sqrt{2}}{10} - \frac{4\sqrt{5}}{10}$$

We now have a common denominator and can finish the problem.

$$\frac{15\sqrt{2}}{10} - \frac{4\sqrt{5}}{10} = \frac{15\sqrt{2} - 4\sqrt{5}}{10}$$

▶ ***Quick check*** Add $\dfrac{1}{\sqrt{5}} + \dfrac{1}{5}$.

Mastery points

Can you

- Identify like radicals?
- Add and subtract like radicals?
- Add and subtract fractions containing radicals?

Exercise 5–5

Perform the indicated operations and simplify. Assume that all variables represent positive real numbers. See examples 5–5 A, B, and C.

Example $3\sqrt{3} + \sqrt{12}$

Solution
$$= 3\sqrt{3} + \sqrt{4 \cdot 3} \qquad \text{Look for factors that are squares}$$
$$= 3\sqrt{3} + 2\sqrt{3} \qquad \text{Product property}$$
$$= (3 + 2)\sqrt{3} = 5\sqrt{3} \qquad \text{Combine like radicals}$$

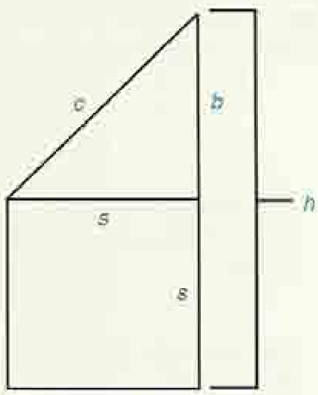
Example $\dfrac{1}{\sqrt{5}} + \dfrac{1}{5}$

Solution
$$= \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} + \frac{1}{5} = \frac{\sqrt{5}}{\sqrt{25}} + \frac{1}{5} = \frac{\sqrt{5}}{5} + \frac{1}{5} \qquad \text{Rationalize the denominator}$$
$$= \frac{\sqrt{5} + 1}{5} \qquad \text{Common denominator, add fractions}$$

1. $7\sqrt{5} + 4\sqrt{5}$
2. $9\sqrt{11} - 2\sqrt{11}$
3. $3\sqrt{3} + 4\sqrt{3}$
4. $10\sqrt{6} - 6\sqrt{6}$
5. $5\sqrt{5} + 7\sqrt{5} - 4\sqrt{5}$
6. $6\sqrt{2} - 4\sqrt{2} + 8\sqrt{2}$
7. $\sqrt{10} + 4\sqrt{10} - 6\sqrt{10}$
8. $8\sqrt{13} - 11\sqrt{13} - 4\sqrt{13}$
9. $5\sqrt[3]{4} + 2\sqrt[3]{4}$
10. $8\sqrt[5]{3} - 4\sqrt[5]{3} + 7\sqrt[5]{3}$
11. $10\sqrt[4]{3} + \sqrt[4]{3} - 5\sqrt[4]{3}$
12. $7\sqrt[3]{11} - 3\sqrt[3]{7} + 2\sqrt[3]{11}$
13. $\sqrt[5]{12} - \sqrt[5]{16} + 4\sqrt[5]{12}$
14. $\sqrt{2a} - 3\sqrt{a} + 4\sqrt{a}$
15. $2\sqrt{3x} - 4\sqrt{2x} + 2\sqrt{3x}$
16. $\sqrt{12} + 4\sqrt{3}$
17. $\sqrt{20} - 3\sqrt{5}$
18. $\sqrt{8} - 4\sqrt{2}$
19. $\sqrt{12} - \sqrt{75}$
20. $2\sqrt{48} - 3\sqrt{27}$
21. $5\sqrt{3} + 4\sqrt{12}$
22. $4\sqrt{7} - 5\sqrt{63}$
23. $2\sqrt{3} + \sqrt{27} - 2\sqrt{12}$
24. $2\sqrt{8} - \sqrt{50} + 5\sqrt{2}$
25. $\sqrt[3]{16} + \sqrt[3]{54}$
26. $\sqrt[3]{81} - \sqrt[3]{24}$
27. $3\sqrt[3]{16} + 5\sqrt[3]{24}$
28. $4\sqrt[3]{54} - 7\sqrt[3]{16}$
29. $\sqrt[3]{81} + 2\sqrt[3]{250}$
30. $\sqrt{50x} + \sqrt{8x}$
31. $\sqrt{32x} - \sqrt{18x}$
32. $4\sqrt{9x} - 5\sqrt{4x}$
33. $6\sqrt{4a^2b} + 5\sqrt{25a^2b}$
34. $-3\sqrt{8a} - 4\sqrt{50a} + 10\sqrt{2a}$
35. $7\sqrt{36a^2b} + 4\sqrt{49a^2b} - 11\sqrt{2b}$
36. $\sqrt[3]{8x^2} - \sqrt[3]{27x^2}$
37. $\sqrt[4]{16a} + \sqrt[4]{81a}$
38. $\sqrt[4]{256b^3} - \sqrt[4]{81b^3}$
39. $-5\sqrt[3]{27a^2} - 4\sqrt[3]{8a^2}$
40. $\sqrt[3]{64x^2y} - 2\sqrt[3]{27x^2y}$
41. $\sqrt[3]{a^6b} + 3a^2\sqrt[3]{b}$
42. $2\sqrt{x^3y} + 5x\sqrt{xy}$
43. $3a^2\sqrt{ab} - a\sqrt{a^3b}$
44. $4xy^2\sqrt{x^3y} + 2x^2y\sqrt{xy^3}$
45. $3a^2b\sqrt{ab^3} - ab^2\sqrt{a^3b}$
46. $\dfrac{1}{2} + \dfrac{1}{\sqrt{2}}$
47. $\dfrac{1}{5} + \dfrac{2}{\sqrt{5}}$
48. $\dfrac{3}{4} + \dfrac{5}{\sqrt{2}}$
49. $\dfrac{4}{9} - \dfrac{2}{\sqrt{3}}$
50. $\dfrac{1}{\sqrt{3}} + \dfrac{1}{\sqrt{5}}$
51. $\dfrac{2}{\sqrt{5}} + \dfrac{3}{\sqrt{6}}$
52. $\dfrac{4}{\sqrt{7}} - \dfrac{2}{\sqrt{14}}$
53. $\dfrac{6}{\sqrt{5}} + \dfrac{2}{\sqrt{10}}$
54. $\dfrac{5}{\sqrt{3}} + \dfrac{2}{\sqrt{6}}$
55. $\dfrac{4}{\sqrt{12}} - \dfrac{1}{\sqrt{48}}$
56. $\dfrac{2}{\sqrt{18}} - \dfrac{4}{\sqrt{50}}$
57. $\dfrac{7}{\sqrt{4x}} - \dfrac{3}{\sqrt{x}}$
58. $\dfrac{5}{\sqrt{9a}} + \dfrac{2}{\sqrt{a}}$
59. $\dfrac{5}{\sqrt{xy}} - \dfrac{4}{\sqrt{x}}$
60. $\dfrac{2}{\sqrt{a}} + \dfrac{3}{\sqrt{ab}}$

Solve the following word problems.

61. We can find the height h of the given figure by finding b from the formula $b = \sqrt{c^2 - s^2}$. If $c = 13$ units and $s = 5$ units, find h.

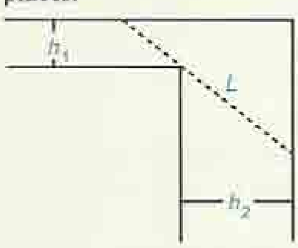


62. Use exercise 61 to find the height of the figure if $c = 10$ feet and $s = 6$ feet.

63. If two hallways of widths h_1 and h_2 meet at right angles, the longest board L that can be carried horizontally around the corner is given by the formula

$$L = \sqrt{(\sqrt[3]{h_1^2} + \sqrt[3]{h_2^2})^3}$$

If h_1 is 8 ft and h_2 is 27 ft, find L. Leave the answer in radical form and also rounded to two decimal places.



64. Use exercise 63 to find L if $h_1 = 6.859$ ft and $h_2 = 21.952$ ft. Leave the answer in radical form and also rounded to two decimal places.

65. The ideal keel length L (in feet) for a hang glider weighing 60 pounds with a pilot weighing P pounds is given by

$$L = \sqrt{\sqrt{2}(60 + P)}$$

If the pilot weighs 175 pounds, find the ideal keel length. Round to two decimal places.

Review exercises

Perform the indicated operations. See section 3–2.

1. $3a(2a - 4)$	2. $(3x - y)(2x + y)$	3. $(a - b)^2$	4. $(a + b)^2$
5. $(2a + b)^2$	6. $(x - y)(x + y)$	7. $(3a - b)(3a + b)$	8. $(4x + 3y)(4x - 3y)$

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5–6 ■ Further operations with radicals

Multiplying radicals

In section 5–3, we learned the procedure for multiplying two radicals. We now combine those ideas along with the **distributive property,** $a(b + c) = ab + ac$, to perform multiplication of radical expressions that contain more than one term.

■ Example 5–6 A

Perform the indicated operations and simplify.

1. $\sqrt{2}(5 + \sqrt{2})$

$= \sqrt{2} \cdot 5 + \sqrt{2} \cdot \sqrt{2}$ Apply distributive property

$= 5\sqrt{2} + \sqrt{4}$ Look for factors that are squares

$= 5\sqrt{2} + 2$ Simplify

2. $\sqrt{3}(\sqrt{15} - \sqrt{21})$

$= \sqrt{3}\sqrt{15} - \sqrt{3}\sqrt{21}$ Apply distributive property

$= \sqrt{45} - \sqrt{63}$ Multiply

$= \sqrt{9 \cdot 5} - \sqrt{9 \cdot 7}$ Look for factors that are squares

$= 3\sqrt{5} - 3\sqrt{7}$ Simplify

3. $(\sqrt{2} + \sqrt{3})(\sqrt{2} + 5\sqrt{3})$

Note In this example, we are multiplying groups together. Therefore, as we did in chapter 3, we will *multiply each term in the first set of parentheses with each term in the second set of parentheses.*

$= \sqrt{2}\sqrt{2} + \sqrt{2} \cdot 5\sqrt{3} + \sqrt{3}\sqrt{2} + \sqrt{3} \cdot 5\sqrt{3}$ Distributive property

$= \sqrt{4} + 5\sqrt{6} + \sqrt{6} + 5\sqrt{9}$ Multiply

$= 2 + 5\sqrt{6} + \sqrt{6} + 5 \cdot 3$ Look for factors that are squares

$= 2 + 5\sqrt{6} + \sqrt{6} + 15$ Multiply

$= (2 + 15) + (5\sqrt{6} + \sqrt{6})$ Combine like terms

$= 17 + 6\sqrt{6}$

4. $(3 - \sqrt{2})(3 + \sqrt{2})$

$= 3 \cdot 3 + 3\sqrt{2} - 3\sqrt{2} - \sqrt{2}\sqrt{2}$ Apply distributive property

$= 9 + 3\sqrt{2} - 3\sqrt{2} - \sqrt{4}$ Look for factors that are squares

$= 9 + 3\sqrt{2} - 3\sqrt{2} - 2$

$= 7 + 0$ Combine like terms

$= 7$

We observe that when we added and subtracted the like terms, there were no longer any radicals in the problem.

▶ ***Quick check*** Perform the indicated operations and simplify.
$\sqrt{2}(\sqrt{6} - \sqrt{10})$ ■

Conjugate factors and rationalizing denominators

In example 4, we see that there are no radicals in the final answer. The type of factors that we are multiplying, called **conjugate factors,** are derived from the factorization of the special product, called the difference of two squares $[(a + b)(a - b) = a^2 - b^2]$. Conjugate factors are used to rationalize the

denominator of a fraction when the denominator has two terms where one or both terms contain a square root. When multiplying conjugate factors, we can simply write our answer as the square of the second term subtracted from the square of the first term.

If we recognize that we are multiplying the factors of the difference of two squares in example 4, the multiplication can be performed as follows:

$$(3 - \sqrt{2})(3 + \sqrt{2}) = (3)^2 - (\sqrt{2})^2 = 9 - 2 = 7$$

We observe that when we combined the like terms, there were no longer any radicals in the answer. Because of this fact, we use the following procedure to rationalize a denominator with two terms where at least one of the terms contains a square root.

Rationalizing a denominator that contains square roots and has two terms

Multiply the numerator and the denominator by the conjugate of the denominator.

To determine the conjugate of a given factor, we write the two terms of the factor and change the sign between them, that is, addition to subtraction, or subtraction to addition.

■ Example 5–6 B

Form the conjugate of the given expressions.

1. $5 - 2\sqrt{3}$ The conjugate is $5 + 2\sqrt{3}$
2. $\sqrt{a} + \sqrt{b}$ The conjugate is $\sqrt{a} - \sqrt{b}$
3. $-3 - \sqrt{7}$ The conjugate is $-3 + \sqrt{7}$ ■

If we wish to rationalize the denominator of the fraction

$$\frac{1}{3 - \sqrt{2}}$$

we recall from example 5–6 A, example 4 that when we multiplied $3 - \sqrt{2}$ by $3 + \sqrt{2}$, there were no radicals left in our product, and this is precisely what we want to occur in our denominator. Therefore to rationalize this fraction, we apply the fundamental principle of fractions and multiply the numerator and the denominator by $3 + \sqrt{2}$, which is the conjugate of the denominator.

$$\begin{aligned}\frac{1}{3 - \sqrt{2}} &= \frac{1}{3 - \sqrt{2}} \cdot \frac{3 + \sqrt{2}}{3 + \sqrt{2}} \\ &= \frac{1(3 + \sqrt{2})}{(3)^2 - (\sqrt{2})^2} \\ &= \frac{3 + \sqrt{2}}{9 - 2} \\ &= \frac{3 + \sqrt{2}}{7}\end{aligned}$$

■ Example 5–6 C

Simplify the following by rationalizing the denominator. Assume that all variables represent positive real numbers, and that no denominator is equal to zero.

1. $\dfrac{5}{\sqrt{13}-\sqrt{3}}$ The conjugate is $\sqrt{13}+\sqrt{3}$

$$= \frac{5}{\sqrt{13}-\sqrt{3}} \cdot \frac{\sqrt{13}+\sqrt{3}}{\sqrt{13}+\sqrt{3}}$$ Use the special product

$$= \frac{5(\sqrt{13}+\sqrt{3})}{(\sqrt{13})^2-(\sqrt{3})^2} = \frac{5(\sqrt{13}+\sqrt{3})}{13-3} = \frac{5(\sqrt{13}+\sqrt{3})}{10}$$

There is a common factor of 5 and we can reduce the fraction.

$$= \frac{\cancel{5}(\sqrt{13}+\sqrt{3})}{\cancel{5}\cdot 2} = \frac{\sqrt{13}+\sqrt{3}}{2}$$

2. $\dfrac{\sqrt{3}}{5-2\sqrt{3}}$ The conjugate is $5+2\sqrt{3}$

$$= \frac{\sqrt{3}}{5-2\sqrt{3}} \cdot \frac{5+2\sqrt{3}}{5+2\sqrt{3}}$$ Use the special product

$$= \frac{\sqrt{3}(5+2\sqrt{3})}{(5)^2-(2\sqrt{3})^2} = \frac{\sqrt{3}(5+2\sqrt{3})}{25-2^2(\sqrt{3})^2} = \frac{\sqrt{3}(5+2\sqrt{3})}{25-4\cdot 3}$$

$$= \frac{\sqrt{3}(5+2\sqrt{3})}{25-12} = \frac{\sqrt{3}(5+2\sqrt{3})}{13}$$ Multiply in the numerator

$$= \frac{5\sqrt{3}+2\sqrt{9}}{13} = \frac{5\sqrt{3}+2\cdot 3}{13} = \frac{5\sqrt{3}+6}{13}$$

3. $\dfrac{\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$ The conjugate is $\sqrt{a}-\sqrt{b}$

$$= \frac{\sqrt{ab}}{\sqrt{a}+\sqrt{b}} \cdot \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$$ Use the special product

$$= \frac{\sqrt{ab}(\sqrt{a}-\sqrt{b})}{(\sqrt{a})^2-(\sqrt{b})^2} = \frac{\sqrt{ab}(\sqrt{a}-\sqrt{b})}{a-b}$$ Multiply in numerator and simplify

$$= \frac{\sqrt{a^2b}-\sqrt{ab^2}}{a-b} = \frac{a\sqrt{b}-b\sqrt{a}}{a-b}$$

▶ ***Quick check*** Simplify $\dfrac{3}{\sqrt{15}-3}$ by rationalizing the denominator. ■

Mastery points

Can you

- Multiply radical expressions containing more than one term?
- Form conjugate factors?
- Multiply conjugate factors?
- Rationalize a denominator that has two terms in which one or both terms contain a square root?

Exercise 5-6

Perform the indicated operations and simplify. Assume that all variables represent positive real numbers and no denominator is equal to zero. See examples 5-6 A, B, and C.

Examples $\sqrt{2}(\sqrt{6} - \sqrt{10})$

Solutions
$= \sqrt{2}\sqrt{6} - \sqrt{2}\sqrt{10}$ Distributive property
$= \sqrt{12} - \sqrt{20}$ Simplify
$= \sqrt{4 \cdot 3} - \sqrt{4 \cdot 5}$ Look for factors that are squares
$= 2\sqrt{3} - 2\sqrt{5}$ Simplify radicals

Examples $\dfrac{3}{\sqrt{15} - 3}$

Solutions
$= \dfrac{3}{\sqrt{15} - 3} \cdot \dfrac{\sqrt{15} + 3}{\sqrt{15} + 3}$ The conjugate is $\sqrt{15} + 3$
$= \dfrac{3(\sqrt{15} + 3)}{(\sqrt{15})^2 - (3)^2} = \dfrac{3(\sqrt{15} + 3)}{15 - 9}$ Use the special products property
$= \dfrac{3(\sqrt{15} + 3)}{6} = \dfrac{\sqrt{15} + 3}{2}$ Common factor of 3, reduce the fraction

1. $3(\sqrt{5} + \sqrt{3})$
2. $5(\sqrt{2} - \sqrt{3})$
3. $4(3\sqrt{7} + \sqrt{2})$
4. $\sqrt{5}(2\sqrt{11} - 3\sqrt{7})$
5. $\sqrt{3}(\sqrt{2} + \sqrt{5})$
6. $\sqrt{6}(\sqrt{7} - \sqrt{5})$
7. $\sqrt{2}(3\sqrt{3} - \sqrt{11})$
8. $2\sqrt{3}(3\sqrt{2} + 4\sqrt{5})$
9. $5\sqrt{5}(7\sqrt{2} - 4\sqrt{3})$
10. $\sqrt{6}(\sqrt{2} + \sqrt{3})$
11. $\sqrt{5}(\sqrt{15} - \sqrt{20})$
12. $\sqrt{14}(\sqrt{35} + \sqrt{10})$
13. $2\sqrt{7}(\sqrt{35} - 4\sqrt{14})$
14. $5\sqrt{3}(2\sqrt{3} - \sqrt{15})$
15. $\sqrt{x}(\sqrt{x} + \sqrt{y})$
16. $\sqrt{a}(\sqrt{ab} - \sqrt{b})$
17. $3\sqrt{x}(2\sqrt{xy} - 5\sqrt{x})$
18. $6\sqrt{ab}(2\sqrt{a} - 3\sqrt{b})$
19. $5\sqrt{xy}(\sqrt{x} + 4\sqrt{y})$
20. $(5 + \sqrt{2})(3 - \sqrt{2})$
21. $(3 + \sqrt{3})(2 + \sqrt{3})$
22. $(6 - \sqrt{6})(6 - \sqrt{6})$
23. $(4 + \sqrt{x})(5 + \sqrt{x})$
24. $(3 - 4\sqrt{x})(4 - 2\sqrt{x})$
25. $(1 + 2\sqrt{y})(3 - 4\sqrt{y})$
26. $(-3 + 5\sqrt{a})(-2 - \sqrt{a})$
27. $(4 - \sqrt{2})(4 + \sqrt{2})$
28. $(4 - \sqrt{5})(4 + \sqrt{5})$
29. $(3 - 2\sqrt{3})(3 + 2\sqrt{3})$
30. $(5 + 4\sqrt{5})(5 - 4\sqrt{5})$
31. $(\sqrt{5} - \sqrt{7})(\sqrt{5} + \sqrt{7})$
32. $(\sqrt{10} - \sqrt{5})(\sqrt{10} + \sqrt{5})$
33. $(\sqrt{a} - b)(\sqrt{a} + b)$
34. $(2\sqrt{a} - \sqrt{b})(2\sqrt{a} + \sqrt{b})$
35. $(3\sqrt{x} - 4\sqrt{y})(3\sqrt{x} + 4\sqrt{y})$
36. $(3 + \sqrt{5})^2$
37. $(2 - \sqrt{7})^2$
38. $(4\sqrt{3} + 2)^2$
39. $(5\sqrt{3} - 4)^2$
40. $(\sqrt{x} - 2y)^2$
41. $(2\sqrt{a} + b)^2$
42. $(3\sqrt{a} + 2\sqrt{b})(\sqrt{a} - 3\sqrt{b})$
43. $(4\sqrt{x} - \sqrt{y})(5\sqrt{x} + \sqrt{y})$
44. $(2\sqrt{a} + \sqrt{b})(3\sqrt{a} - \sqrt{b})$
45. $\dfrac{1}{\sqrt{3} + 2}$
46. $\dfrac{1}{\sqrt{5} - 2}$
47. $\dfrac{6}{4 + \sqrt{6}}$
48. $\dfrac{6}{3 - \sqrt{6}}$
49. $\dfrac{4}{\sqrt{10} - \sqrt{6}}$
50. $\dfrac{5}{\sqrt{11} - \sqrt{6}}$
51. $\dfrac{3}{4 - 2\sqrt{3}}$
52. $\dfrac{5}{6 - 2\sqrt{5}}$
53. $\dfrac{\sqrt{6}}{\sqrt{2} - 2\sqrt{3}}$
54. $\dfrac{\sqrt{10}}{\sqrt{5} + 2\sqrt{2}}$
55. $\dfrac{\sqrt{14}}{3\sqrt{7} - 2\sqrt{2}}$
56. $\dfrac{\sqrt{2}}{3\sqrt{6} - \sqrt{3}}$
57. $\dfrac{\sqrt{x}}{\sqrt{x} + \sqrt{y}}$
58. $\dfrac{\sqrt{a}}{\sqrt{a} + \sqrt{b}}$
59. $\dfrac{\sqrt{ab}}{\sqrt{ab} + \sqrt{b}}$
60. $\dfrac{\sqrt{a} + b}{\sqrt{a} - b}$

61. $\dfrac{\sqrt{x} - y}{\sqrt{x} + y}$

62. $\dfrac{a + \sqrt{b}}{a - \sqrt{b}}$

63. $\dfrac{a}{\sqrt{ab} - \sqrt{a}}$

64. $\dfrac{x^2y}{x\sqrt{x} - \sqrt{xy}}$

65. $\dfrac{2a}{\sqrt{a} - \sqrt{ab}}$

66. $\dfrac{3x}{\sqrt{x} - \sqrt{xy}}$

67. $\dfrac{\sqrt{a}}{\sqrt{ab} - \sqrt{a}}$

68. $\dfrac{2\sqrt{x}}{3\sqrt{x} - 2\sqrt{y}}$

69. The electric-field intensity on the axis of a uniform charged ring is given by

$$E = \frac{T}{(x^2 + r^2)^{3/2}}$$

where T is the total charge on the ring and r is the radius of the ring. Express the rational exponent in radical form and leave the answer in standard form.

Review exercises

Perform the indicated operations. See section 3–2.

1. $(2a + b)(a - b)$
2. $(a - 2b)(a - b)$
3. $(2a + 3b)(2a - 3b)$
4. $(a + 3)^2$

Simplify the following. See section 5–3.

5. $\sqrt{16}$
6. $\sqrt{20}$
7. $\sqrt{6}\sqrt{6}$
8. $\sqrt{24}\sqrt{3}$

5–7 ■ Complex numbers

Imaginary numbers

In this section, we will examine what happens when we try to take the square root of a negative number. The expression $\sqrt{-4}$ has no meaning in the system of real numbers because there is no real number that when multiplied by itself equals a negative number, in this case, -4. However there are situations in which an answer for such a problem is required. For example, we need to find the square root of a negative number if we want to find the solution to the equation $x^2 + 4 = 0$. In electronics, the impedance of a circuit, which is the total effective resistance to the flow of current caused by a combination of elements in the circuit, requires that we find the square root of a negative number. We define a new number to provide the required result.

Definition of *i*

The number i is a number such that

$$i = \sqrt{-1}$$

and

$$i^2 = -1$$

We can use this definition to define the square root of any negative number.

Definition of $\sqrt{-b}$

For any positive real number b, we define
$$\sqrt{-b} = i\sqrt{b}$$

We now define the system of imaginary numbers as the set of all numbers that can be expressed in the form bi, where b is an element of the set of real numbers.

Example 5–7 A

Simplify the following.

1. $\sqrt{-4} = i\sqrt{4}$ — The first step is always to rewrite $\sqrt{-b}$ as $i\sqrt{b}$
   $= 2i$ — Simplify the radical
2. $\sqrt{-2} = i\sqrt{2}$ — Rewrite $\sqrt{-b}$ as $i\sqrt{b}$, the $\sqrt{2}$ will not simplify

Note Whenever we are dealing with the square root of a negative number, we must express our problem in terms of i before proceeding.

If we wish to check our results, we can square the answer to get the original radicand back.

Example 5–7 B

Simplify the following.

1. $(2i)^2 = 2^2 \cdot i^2$ — Square each factor
   $= 4 \cdot (-1)$ — i^2 is replaced with -1
   $= -4$ — Simplify
2. $(\sqrt{2}\,i)^2 = (\sqrt{2})^2 \cdot i^2$ — Square each factor
   $= 2(-1) = -2$ — Replace i^2 with -1 and simplify
3. $\sqrt{-2}\sqrt{-8} = i\sqrt{2} \cdot i\sqrt{8}$ — Rewrite $\sqrt{-b}$ as $i\sqrt{b}$
   $= i^2\sqrt{16}$ — Multiply $i \cdot i$ and multiply radicals
   $= (-1)(4) = -4$ — Replace i^2 with -1 and simplify
4. $\sqrt{-6}\sqrt{-3} = i\sqrt{6} \cdot i\sqrt{3}$ — Rewrite $\sqrt{-b}$ as $i\sqrt{b}$
   $= i^2\sqrt{18}$ — Multiply $i \cdot i$ and multiply radicals
   $= (-1) \cdot 3\sqrt{2} = -3\sqrt{2}$ — Replace i^2 with -1 and simplify radical

▶ ***Quick check*** Simplify. $\sqrt{-5}\sqrt{-15}$

If we apply the properties of exponents to different exponents of i, we have

$$\text{A cycle}\begin{cases} i = i \\ i^2 = -1 \\ i^3 = i^2 i = (-1)i = -i \\ i^4 = i^2 i^2 = (-1)(-1) = 1 \end{cases} \qquad \text{A cycle}\begin{cases} i^5 = i^4 i = 1 \cdot i = i \\ i^6 = i^4 i^2 = 1(-1) = -1 \\ i^7 = i^4 i^3 = 1(-i) = -i \\ i^8 = i^4 i^4 = 1 \cdot 1 = 1 \end{cases}$$

It can be seen that the powers of i go through the cycle of i, -1, $-i$, and 1. Using this fact, it is possible to simplify i raised to any positive integer power.

■ Example 5–7 C

Simplify.

1. $i^{10} = i^4 \cdot i^4 \cdot i^2 = 1 \cdot 1 \cdot (-1) = -1$
2. $i^{15} = i^4 \cdot i^4 \cdot i^4 \cdot i^3 = 1 \cdot 1 \cdot 1 \cdot (-i) = -i$
3. $i^{20} = i^4 i^4 i^4 i^4 i^4 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$

Replace i^4 with 1
Replace i^3 with $-i$
Replace i^2 with -1 ■

From these examples, we can see that **when we simplify i to a positive integer power, the resulting power of i is the remainder when we divide the original power by 4.**

■ Example 5–7 D

Simplify.

1. $i^{50} = i^2 = -1$ because $50 \div 4 = 12$ Remainder 2
2. $i^{79} = i^3 = -i$ because $79 \div 4 = 19$ Remainder 3
3. $i^{21} = i$ because $21 \div 4 = 5$ Remainder 1 ■

Complex numbers

Now let us define a new type of number that combines the system of real numbers and the system of imaginary numbers. These new numbers are called **complex numbers** and are composed of a real part denoted by a and an imaginary part denoted by b.

Definition of a complex number

A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and i represents $\sqrt{-1}$.

$$\underbrace{a}_{\text{Real part}} + \underbrace{b}_{\text{Imaginary part}}\, i \longleftarrow \sqrt{-1}$$

$a + bi$ is called the **standard form** of a complex number.

If $a = 0$, $\quad 0 + bi = bi$ $\quad$ Imaginary number
If $b = 0$, $\quad a + 0i = a$ $\quad$ Real number

Figure 5–1 shows the relationship among the various sets of numbers that we have studied.

Operations with complex numbers

The commutative, associative, and distributive properties for real numbers are also valid for complex numbers. If we wish to perform addition and subtraction of complex numbers, we do so by combining the real parts and combining the imaginary parts.

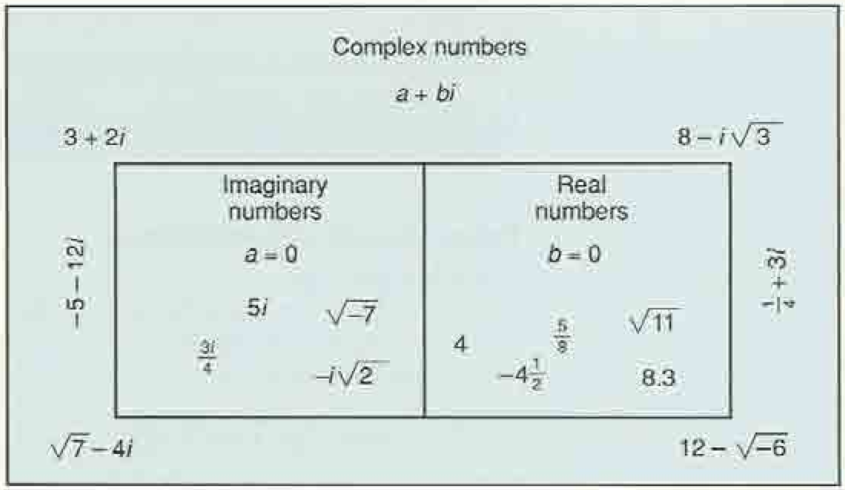

Figure 5–1

Addition or subtraction of complex numbers

1. Combine the real parts.
2. Combine the imaginary parts.
3. Leave the result in the form $a + bi$.

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$
$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

Example 5–7 E

Perform the indicated addition and subtraction.

1. $(4 + 5i) + (2 + 3i)$
To perform the addition, we add the real parts $(4 + 2)$ and the imaginary parts $(5 + 3)$.

$$= (4 + 2) + (5 + 3)i$$
$$= 6 + 8i$$

2. $(6 + 11i) - (5 + 2i)$
To perform the subtraction, we subtract the real parts $(6 - 5)$ and the imaginary parts $(11 - 2)$.

$$= (6 - 5) + (11 - 2)i$$
$$= 1 + 9i$$

▶ ***Quick check*** Add $(7 + 6i) + (1 + 2i)$

When we are multiplying two complex numbers such as

$$(a + bi)(c + di)$$

we multiply each term in the first parentheses with each term in the second parentheses.

Multiplication of two complex numbers

1. Multiply the numbers as if they are two binomials.
2. Substitute -1 for i^2.
3. Combine the like terms and leave the result in the form $a + bi$.

Example 5–7 F

Perform the indicated multiplication.

1. $(2 + 3i)(5 + 2i)$

$= 2 \cdot 5 + 2 \cdot 2i + 3i \cdot 5 + 3i \cdot 2i$	Distribute the multiplication
$= 10 + 4i + 15i + 6i^2$	Simplify
$= 10 + 4i + 15i - 6$	Replace i^2 with -1, then $6i^2 = 6(-1) = -6$
$= (10 - 6) + (4 + 15)i$	Combine the real parts, combine the imaginary parts
$= 4 + 19i$	Standard form

2. $(3 - 4i)(3 + 4i)$

$= 3 \cdot 3 + 3 \cdot 4i - 4i \cdot 3 - 4i \cdot 4i$	Distribute the multiplication
$= 9 + 12i - 12i - 16i^2$	Simplify
$= 9 + 12i - 12i + 16$	Replace i^2 with -1, then $-16i^2 = (-16)(-1) = 16$
$= (9 + 16) + (12 - 12)i$	Combine the real parts, combine the imaginary parts
$= 25 + 0i$	The imaginary part is 0
$= 25$	The product is a real number

▶ ***Quick check*** Multiply $(1 + 2i)(6 + 3i)$

The factors $(3 - 4i)$ and $(3 + 4i)$ are called **complex conjugates** and their product will be a real number. We use complex conjugates to find the quotient of two complex numbers. Consider the quotient

$$(2 + 3i) \div (3 - 4i) = \frac{2 + 3i}{3 - 4i}$$

We would like to perform the division of the two complex numbers and leave the answer in the standard form $a + bi$. First of all, we will eliminate the i in the denominator, since this is just another form of the radical $\sqrt{-1}$. To rationalize the denominator, we multiply by the conjugate of $3 - 4i$, which is $3 + 4i$.

$$\frac{2 + 3i}{3 - 4i} \cdot \frac{3 + 4i}{3 + 4i} = \frac{2 \cdot 3 + 2 \cdot 4i + 3i \cdot 3 + 3i \cdot 4i}{(3)^2 - (4i)^2}$$

$$= \frac{6 + 8i + 9i + 12i^2}{9 - 16i^2}$$

Replacing i^2 with -1, $12i^2$ becomes -12 and $-16i^2$ becomes 16.

$$= \frac{6 + 8i + 9i - 12}{9 + 16}$$

Adding the like terms, we have

$$= \frac{-6 + 17i}{25}$$

Since the answer is to be stated in the form $a + bi$, we divide each term in the numerator by 25 to obtain

$$= \frac{-6}{25} + \frac{17}{25}i$$

This is the quotient of the two complex numbers stated in standard form.

Division of one complex number by another complex number

1. Write the division as a fraction.
2. Multiply the numerator and the denominator by the conjugate of the denominator.
3. Multiply and simplify in the numerator. Use the special product property to simplify the denominator to a real number.
4. Write the result in the form $a + bi$.

Example 5–7 G

Perform the indicated division.

1. $\dfrac{1 + \sqrt{-4}}{3 - \sqrt{-9}}$

We must first simplify the radicals before carrying out the division.

$$= \frac{1 + i\sqrt{4}}{3 - i\sqrt{9}} = \frac{1 + 2i}{3 - 3i}$$

The conjugate of the denominator is $3 + 3i$

$$= \frac{1 + 2i}{3 - 3i} \cdot \frac{3 + 3i}{3 + 3i} = \frac{3 + 3i + 6i + 6i^2}{(3)^2 - (3i)^2}$$

$$= \frac{3 + 3i + 6i - 6}{9 - 9i^2} = \frac{-3 + 9i}{9 + 9} = \frac{-3 + 9i}{18}$$

$$= \frac{-3}{18} + \frac{9}{18}i = \frac{-1}{6} + \frac{1}{2}i$$

2. $\dfrac{-3 - 2i}{i} = \dfrac{-3 - 2i}{i} \cdot \dfrac{-i}{-i}$

The denominator is an imaginary number and can be written as $0 + i$. Its conjugate is $0 - i$ or just $-i$.

$$= \frac{3i + 2i^2}{-i^2} = \frac{3i - 2}{1}$$

Multiply and simplify

$$= \frac{3i}{1} - \frac{2}{1} = 3i - 2 = -2 + 3i$$

Standard form ■

Mastery points

Can you

- Simplify the square root of a negative number?
- Simplify i raised to a positive integer power?
- Add, subtract, multiply, and divide complex numbers?

Exercise 5–7

Simplify the following. See examples 5–7 A, B, C, and D.

Example $\sqrt{-5}\sqrt{-15}$

Solution $= i\sqrt{5} \cdot i\sqrt{15}$
$= i^2\sqrt{75}$
$= (-1)5\sqrt{3}$
$= -5\sqrt{3}$

1. $\sqrt{-9}$
2. $\sqrt{-16}$
3. $\sqrt{-12}$
4. $\sqrt{-18}$
5. $(3i)^2$
6. $(4i)^2$
7. $(\sqrt{3}\,i)^2$
8. $(\sqrt{3}\,i)^2$
9. $\sqrt{-3}\sqrt{-5}$
10. $\sqrt{-7}\sqrt{-11}$
11. $\sqrt{-2}\sqrt{-2}$
12. $\sqrt{-6}\sqrt{-6}$
13. $(\sqrt{-5})^2$
14. $(\sqrt{-4})^2$
15. $\sqrt{-3}\sqrt{-12}$
16. $\sqrt{-2}\sqrt{-18}$
17. $\sqrt{-3}\sqrt{-15}$
18. $\sqrt{-7}\sqrt{-14}$
19. i^{10}
20. i^{15}
21. i^{44}
22. i^{27}
23. i^{19}
24. i^{60}

Perform the indicated operations and leave the answer in standard form. See examples 5–7 E, F, and G.

Example $(7 + 6i) + (1 + 2i)$

Solution $= (7 + 1) + (6 + 2)i$ Add the real parts, add the imaginary parts
$= 8 + 8i$ Standard form

Example $(1 + 2i)(6 + 3i)$

Solution $= 1 \cdot 6 + 1 \cdot 3i + 2i \cdot 6 + 2i \cdot 3i$ Distributive property
$= 6 + 3i + 12i + 6i^2$ Simplify
$= 6 + 3i + 12i - 6$ Replace i^2 with -1, then $6i^2 = 6(-1) = -6$
$= (6 - 6) + (3 + 12)i$ Combine the real parts, combine the imaginary parts
$= 0 + 15i$ The real part is 0
$= 15i$ The product is an imaginary number

25. $(6 + 3i) + (2 + 4i)$
26. $(4 + 3i) + (1 + i)$
27. $(6 - 2i) - (8 - 4i)$
28. $(4 - 5i) - (3 - 7i)$
29. $(2 + \sqrt{-49}) - (1 - \sqrt{-1})$
30. $(9 + \sqrt{-64}) - (9 - \sqrt{-9})$
31. $(4 - \sqrt{-25}) - (4 - \sqrt{-36})$
32. $[(2 + 5i) + (3 - 2i)] + (3 - i)$
33. $[(-2 - i) + (3 + 2i)] + (4 - 3i)$
34. $[(8 - 5i) - (5 + 4i)] + (6 - 7i)$
35. $[(9 - i) - (6 - 4i)] + (5 + 5i)$
36. $(4 + 3i)(1 + i)$
37. $(3 - 2i)(3 + 2i)$
38. $(4 + 5i)(4 - 5i)$
39. $(3 + i)(5 - 4i)$
40. $(2 + \sqrt{-16})(3 - \sqrt{-25})$
41. $(7 + \sqrt{-1})(3 + \sqrt{-4})$
42. $(5 - \sqrt{-25})(4 + \sqrt{-16})$
43. $(5 - \sqrt{-9})(5 + \sqrt{-9})$
44. $(2 + i)^2$
45. $(4 - 3i)^2$
46. $(3 - \sqrt{-9})^2$
47. $(2 + \sqrt{-4})^2$
48. $\dfrac{3 - 2i}{i}$
49. $\dfrac{4 + 5i}{i}$
50. $\dfrac{6 - 2i}{3i}$
51. $\dfrac{2 + 4i}{2i}$
52. $\dfrac{4 - 9i}{\sqrt{-1}}$
53. $\dfrac{5 + 7i}{\sqrt{-9}}$
54. $\dfrac{4 - 3i}{1 + i}$

55. $\frac{5-2i}{5-i}$
56. $\frac{4-5i}{2+i}$
57. $\frac{3-i}{3+i}$
58. $\frac{5-i}{5+i}$
59. $\frac{2+5i}{3-2i}$
60. $\frac{4+3i}{3-i}$
61. $\frac{6+3i}{3+4i}$
62. $\frac{5-\sqrt{-4}}{3+\sqrt{-9}}$
63. $\frac{2+\sqrt{-16}}{4-\sqrt{-1}}$
64. $\frac{7-\sqrt{-25}}{3+\sqrt{-36}}$

Solve the following word problems.

65. The impedance of an electrical circuit is the measure of the total opposition to the flow of an electric current. The impedance Z in a series RCL circuit is given by

$$Z = R + i(X_L - X_C)$$

Determine the impedance if $R = 30$ ohms, $X_L = 16$ ohms, and $X_C = 40$ ohms.

66. Use the formula in exercise 65 to find Z if $R = 28$ ohms, $X_L = 16$ ohms, and $X_C = 38$ ohms.

67. The impedance Z in an AC circuit is given by the formula

$$Z = \frac{V}{I}$$

where V is the voltage and I is the current. Find Z when $V = 0.3 + 1.2i$ and $I = 2.1i$. Round all values to three decimal places.

68. Use the formula in exercise 67 to find Z if $V = 2.2 - 0.3i$ and $I = -1.1i$. Round all values to three decimal places.

69. The total impedance Z_T of an AC circuit containing impedances Z_1 and Z_2 in parallel is given by the formula

$$Z_T = \frac{Z_1Z_2}{Z_1 + Z_2}$$

Find Z_T when $Z_1 = 3 - i$ and $Z_2 = 2 + i$.

70. Use the formula in exercise 69 to find Z_T if $Z_1 = 4 - i$ and $Z_2 = 3 + i$.

71. If three resistors in an AC circuit are connected in parallel, the total impedance Z_T is given by the formula

$$Z_T = \frac{Z_1Z_2Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

Find Z_T when $Z_1 = 3 - i$, $Z_2 = 3 + i$, and $Z_3 = 2i$.

72. Use the formula in exercise 71 to find Z_T if $Z_1 = 2 - i$, $Z_2 = 2 + i$, and $Z_3 = 2i$.

73. For what values of x does the expression $\sqrt{5-x}$ represent a real number?

74. For what values of x does the expression $\sqrt{x+4}$ represent a real number?

75. For what values of x does the expression $\sqrt{x+11}$ represent an imaginary number?

76. For what values of x does the expression $\sqrt{8-x}$ represent an imaginary number?

Review exercises

Factor completely. See section 3–8.

1. $x^2 - 12x + 36$
2. $3x^2 + 9x$
3. $x^2 - 16$
4. $9x^2 - 36$
5. $x^2 - 7x + 10$
6. $2x^2 - x - 3$
7. $5x^2 - 14x - 3$
8. $6x^2 - 23x - 4$

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Chapter 5 lead-in problem

The formula for approximating the velocity V in miles per hour of a car based on the length of its skid marks S (in feet) on wet pavement is given by

$$V = 2\sqrt{3S}$$

If the skid marks are 147 feet long, what was the velocity of the car?

Solution

$V = 2\sqrt{3S}$	Original formula
$V = 2\sqrt{3(\ \ )}$	Formula ready for substitution
$V = 2\sqrt{3(147)}$	Substitute
$V = 2\sqrt{441}$	Simplify under radical
$V = 2 \cdot 21$	Simplify radical
$V = 42$	Multiply

Hence, the velocity of the car was 42 miles per hour.

Chapter 5 summary

1. $a^{1/n} = \sqrt[n]{a}$, whenever the principal n^{th} root of a is a real number.
2. $a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$, if the principal n^{th} root of a is a real number.
3. For all nonnegative real numbers a and b and positive integer n greater than 1,

   $$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$$

   $$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \qquad b \neq 0$$

   and

   $$\sqrt[kn]{a^{km}} = \sqrt[n]{a^m}$$
4. We eliminate radicals from the denominator of a fraction by **rationalizing the denominator.**
5. We can only add or subtract *like* radicals.
6. **Conjugate factors** are used to rationalize the denominator of a fraction when the denominator has two terms where one or both terms contain a square root.
7. We define $i = \sqrt{-1}$, so that $i^2 = -1$.
8. A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and i represents $\sqrt{-1}$.

Chapter 5 error analysis

1. Principal nth root
   Example: $\sqrt{81} = 9$ or -9
   Correct answer: 9
   What error was made? (*see page 216*)
2. Principal nth roots
   Example: $\sqrt[3]{-8}$ does not exist in the real numbers.
   Correct answer: -2
   What error was made? (*see page 216*)
3. Rational number exponents
   Example: $(32)^{-1/5} = -(32)^{1/5} = -2$
   Correct answer: $\frac{1}{2}$
   What error was made? (*see page 221*)
4. Operations with rational exponents
   Example: $2^{1/3} \cdot 2^{1/3} = 2^{1/9}$
   Correct answer: $2^{2/3}$
   What error was made? (*see page 224*)
5. Product of radicals
   Example: $\sqrt[4]{3} \cdot \sqrt[4]{2} = \sqrt[4]{5}$
   Correct answer: $\sqrt[4]{6}$
   What error was made? (*see page 228*)
6. Reducing the index
   Example: $\sqrt[6]{x^3y} = \sqrt{xy}$
   Correct answer: $\sqrt[6]{x^3y}$
   What error was made? (*see page 229*)
7. Rationalizing the denominator
   Example: $\frac{1}{\sqrt[4]{x}} = \frac{\sqrt[4]{x}}{x}$
   Correct answer: $\frac{\sqrt[4]{x^3}}{x}$
   What error was made? (*see page 234*)
8. Sum of radicals
   Example: $\sqrt{5} + \sqrt{5} = \sqrt{10}$
   Correct answer: $2\sqrt{5}$
   What error was made? (*see page 238*)
9. Radical of a sum
   Example: $\sqrt{16 + 9} = \sqrt{16} + \sqrt{9} = 4 + 3 = 7$
   Correct answer: 5
   What error was made? (*see page 238*)
10. Multiplying radicals
    Example: $\sqrt{6}(\sqrt{6} - 3) = 3$
    Correct answer: $6 - 3\sqrt{6}$
    What error was made? (*see page 242*)

Chapter 5 critical thinking

Choose a two-digit number whose one's digit is 5 (such as 35). You can find the square of this number if you multiply the ten's digit by one more than the ten's digits and place 25 after this product. For example:

$$35^2$$

$$\underbrace{3}_{\text{Ten's digit}} \cdot \underbrace{4}_{\text{One more than the ten's digit}} = \underbrace{12}_{\text{Product}}\ \underbrace{25}_{\text{Place 25 after the product}}$$

Therefore, $35^2 = 1225$
Why is this true?

Chapter 5 review

Assume that all variables represent positive real numbers and no denominator is equal to zero.

[5–1]

Rewrite the following in radical notation and use table 5–1 to simplify.

1. $36^{1/2}$ **2.** $16^{-3/4}$ **3.** $(-27)^{2/3}$

[5–2]

Perform the indicated operations and simplify. Leave the answer with all exponents positive.

4. $a^{2/3} \cdot a^{1/4}$ **5.** $(c^{3/4})^{1/3}$ **6.** $(27x^3)^{2/3}$ **7.** $(b^{-2/3})^{-3}$

8. $\dfrac{a^{1/2}}{a^{1/3}}$ **9.** $(8x^{12}y^6)^{2/3}$ **10.** $\dfrac{x^3x^{2/3}}{x^{1/2}}$ **11.** $\dfrac{a^{2/3}}{a^{-1/2}}$

[5–3]

Simplify the following.

12. $\sqrt{12}$ **13.** $\sqrt{10}\,\sqrt{15}$ **14.** $\sqrt[5]{x^4}\,\sqrt[5]{x^3}$

15. $\sqrt[3]{6ab^2}\,\sqrt[3]{4a^2b^2}$ **16.** $\sqrt[4]{a^6}$ **17.** $\sqrt[3]{8a^3b^6}$

18. If the hypotenuse of a right triangle is 10 inches long and one of the legs is 6 inches long, find the length of the other leg.

[5–4]

Simplify the following expressions leaving no radicals in the denominator.

19. $\sqrt{\dfrac{49}{64}}$ **20.** $\sqrt{\dfrac{16}{27}}$ **21.** $\sqrt[3]{\dfrac{16x^3y^2}{z^6}}$ **22.** $\sqrt{\dfrac{1}{8}}$ **23.** $\dfrac{6}{\sqrt{2}}$

24. $\sqrt[3]{\dfrac{8}{25}}$ **25.** $\sqrt{\dfrac{x^2}{y}}$ **26.** $\sqrt[3]{\dfrac{a^2}{b}}$ **27.** $\dfrac{x}{\sqrt[5]{x^2}}$ **28.** $\sqrt[3]{\dfrac{a}{bc^2}}$

29. $\dfrac{a}{\sqrt[4]{ab^2}}$ **30.** $\sqrt{\dfrac{3x}{y^2}}\,\sqrt{\dfrac{xy}{3}}$

[5–5]

Perform the indicated operations and simplify.

31. $4\sqrt{3} - 3\sqrt{3} + 7\sqrt{3}$ **32.** $\sqrt{18} + \sqrt{50}$ **33.** $\sqrt{8a} + 9\sqrt{18a}$

34. $5x^2\sqrt{xy} - 2x\sqrt{x^3y}$ **35.** $\dfrac{5}{\sqrt{6}} - \dfrac{1}{\sqrt{3}}$ **36.** $\dfrac{2}{\sqrt{ab}} - \dfrac{1}{\sqrt{a}}$

[5–6]

Simplify the following expressions leaving no radicals in the denominator.

37. $\sqrt{2}(\sqrt{6} - \sqrt{10})$
38. $2\sqrt{a}(\sqrt{ab} + 2\sqrt{a})$
39. $(5 - \sqrt{5})^2$
40. $(\sqrt{10} - \sqrt{7})(\sqrt{10} + \sqrt{7})$
41. $(2\sqrt{a} + 3\sqrt{b})(2\sqrt{a} - 3\sqrt{b})$
42. $(3\sqrt{x} + y)^2$
43. $\dfrac{1}{\sqrt{6} + 2}$
44. $\dfrac{10}{4 + \sqrt{6}}$
45. $\dfrac{\sqrt{3}}{\sqrt{6} - \sqrt{3}}$
46. $\dfrac{a^2b}{a\sqrt{a} - \sqrt{ab}}$

[5–7]

Simplify the following.

47. $\sqrt{-49}$
48. $\sqrt{-28}$
49. $(2i)^2$
50. $(\sqrt{7}i)^2$
51. $\sqrt{-3}\sqrt{-12}$
52. $(\sqrt{-3})^2$
53. $\sqrt{-2}\sqrt{-3}$
54. i^{37}

Simplify the following and leave the answer in standard form.

55. $(4 + 2i) + (3 + 5i)$
56. $(2 - \sqrt{-36}) - (3 + \sqrt{-25})$
57. $(2 - \sqrt{-9})(3 + \sqrt{-16})$
58. $(2 + 5i)^2$
59. $\dfrac{3 + 4i}{i}$
60. $\dfrac{7 - 6i}{\sqrt{-9}}$
61. $\dfrac{4 - i}{2 + i}$
62. $\dfrac{9 - \sqrt{-4}}{7 - \sqrt{-9}}$

Chapter 5 cumulative test

Factor completely.

[3–5] 1. $a^2 - 7a - 8$
[3–4] 2. $4x^2 - 3x$
[3–7] 3. $9x^2 - 36$
[3–6] 4. $2x^2 + 11x + 12$
[3–6] 5. $3a^2 - 11a - 20$
[3–6] 6. $6x^2 + 17x + 12$
[1–5] 7. Evaluate the expression $b^2 - 4ac$ at
(a) $a = 1$, $b = 4$, and $c = -3$;
(b) $a = 2$, $b = -4$, and $c = 3$.

Find the solution set.

[2–1] 8. $3(2x - 1) + 4 = x + 3$
[2–5] 9. $5x + 7 > 2x - 4$
[2–2] 10. $3x - 2y = 4(x + y)$
[2–4] 11. $|3x - 1| = 4$
[2–6] 12. $|2x + 3| > 8$
[2–1] 13. $x(x + 1) - (x + 3)^2 = 4$
[2–6] 14. $|1 - 4x| \leq 7$

Simplify the following and leave in standard form. Assume that all variables represent positive real numbers.

[5–3] 15. $\sqrt[5]{64a^{10}b^7}$
[5–7] 16. $(3 - 4i)(2 + i)$
[5–3] 17. $\sqrt{48}$
[5–2] 18. $a^{1/3} \cdot a^{1/4}$
[5–3] 19. $\sqrt[3]{8a^4b^6}$
[3–1] 20. $(2a^3b^4c)^3$
[5–7] 21. $\sqrt{-18}$
[5–6] 22. $\dfrac{4}{\sqrt{10} + \sqrt{6}}$
[5–7] 23. $\dfrac{1 - 3i}{2 + 3i}$
[5–2] 24. $(4a^6)^{1/2}$
[3–3] 25. $\dfrac{3x^{-2}y^4}{9x^{-5}y^2}$
[5–4] 26. $\sqrt[3]{\dfrac{a^2}{4bc^2}}$

Solve the following word problems.

[2–3] **27.** When the length of a side of a square is increased by 4 inches, the area is increased by 72 square inches. Find the original length of a side.

[2–3] **28.** A metallurgist wishes to form 1,000 kg of an alloy that is 62% copper. This alloy is to be obtained by fusing some alloy that is 80% copper and some that is 50% copper. How many kilograms of each alloy must be used?

[5–1] **29.** In the theory of ballistics, the ballistic limit v of a material is approximated by the formula

$$v = kT^{6/5}$$

where T is the thickness of a sheet of material and k is a constant that is dependent on the material being used. Compute the ballistic limit if $k = 24{,}000$ and $T = 0.03125$.

[5–1] **30.** Use the formula in exercise 29 to find v if $k = 25{,}000$ and $T = 0.07776$.

30. Inlet pipe fills $\frac{1}{45}$ of the tank in 1 minute

Outlet pipe empties $\frac{1}{30}$ of the tank in 1 minute.

Let x = number of minutes to empty the tank

$$\frac{1}{30} - \frac{1}{45} = \frac{1}{x}$$

$$90x \cdot \frac{1}{30} - 90x \cdot \frac{1}{45} = 90x \cdot \frac{1}{x}$$

$$3x - 2x = 90$$

$$x = 90$$

It would take 90 minutes to empty the tank.

Review exercises

1. $8x^3 - 12x^2 + 4x$ 2. $3x^2 - 16x + 5$ 3. $25z^2 - 40z + 16$

4. $4y^2 - 9$ 5. $\frac{1}{y}$ 6. x^4y^3 7. -7 or 7 8. -5

Chapter 4 review

1. $\{x|x \in R, x \neq -7\}$ 2. $\left\{x|x \in R, x \neq \frac{4}{3}\right\}$

3. $\{x|x \in R, x \neq 5\}$ 4. $\left\{a|a \in R, a \neq -\frac{4}{3}, \frac{4}{3}\right\}$

5. $\left\{z|z \in R, z \neq -\frac{5}{2}, \frac{1}{3}\right\}$ 6. $\left\{y|y \in R, y \neq \frac{2}{3}\right\}$ 7. $\frac{ab^3}{c^2}$

8. $\frac{-2n^2}{7mp^4}$ 9. $\frac{5}{6}$ 10. $\frac{3}{a-2}$ 11. $\frac{-5}{2y+x}$

12. $\frac{y^2+4y+16}{y+4}$ 13. $\frac{a-12}{a+1}$ 14. $\frac{4x+3}{5x-1}$

15. $\frac{-(2y+3)}{2(3y+2)}$ 16. $\frac{6y}{x}(x \neq 0, y \neq 0)$

17. $12ay(a \neq 0, y \neq 0)$ 18. $\frac{(4p+3)(p-4)}{3}\left(p \neq -4, \frac{3}{4}\right)$

19. $\frac{z-3}{2(z+1)(z-1)}(z \neq -1,1,3)$

20. $\frac{(m^2+2m+4)(m+6)}{m(m+5)}(m \neq -5,0,2,3)$

21. $1\left(a \neq -7,-3,-\frac{2}{5},\frac{1}{2}\right)$

22. $\frac{(x+7)(2x-1)}{(4x^2-2x+1)(x+2)}\left(x \neq -2,-\frac{1}{2},\frac{1}{2},7\right)$

23. $\frac{x^2}{(4x+5)^2}\left(x \neq -\frac{5}{4},\frac{5}{4}\right)$ 24. $\frac{y+3}{y+2}\left(y \neq -3,-2,\frac{4}{7}\right)$

25. $\frac{m-n}{m+n}\left(m \neq -n,\frac{n}{2}; q \neq -p,p\right)$ 26. $180x^3y^3$

27. $6x^2(x+2)(x+4)(x-4)$ 28. $3a(a+5)(a-2)$

29. $p(p-5)(p+5)^2$ 30. $\frac{41x}{12y}$ 31. $\frac{-2n^2-28n-25}{(n+4)(n-1)}$

32. $\frac{10p^2+29p+81}{p(p+9)(p+2)(p-2)}$ 33. $\frac{6b^2+16b-23}{3b-2}$

34. $\frac{2y^2+18y+3}{(y+7)(y-7)}$ 35. $\frac{-(4x^2+15x+4)}{(x-6)(x+6)(x+4)}$

36. $\frac{37}{2(a-2)}$ 37. $\frac{-5x^2+63x-102}{8(x-7)(x+4)}$ 38. $\frac{4a+b}{(a-2b)(a+2b)}$

39. $\frac{1}{R_t} = \frac{I_1E_2E_3 + I_2E_1E_3 + I_3E_1E_2}{E_1E_2E_3}$ 40. $\frac{2}{a}$ 41. $\frac{5x}{4x-12}$

42. $\frac{3x+6}{x-5}$ 43. $\frac{7b-38}{8b-34}$ 44. $\frac{x^2y-xy^2}{2y+3x}$

45. $\frac{p^2-4p+5}{p^2-4}$ 46. $5a^6+3a^2+2$ 47. $6a^3b^2c^2-3c^3+1$

48. $3x^2-3x+4$ 49. x^3+x^2-x+1 50. $P(-2) = -35$

51. $P(1) = 11$ 52. $P(-1) = -12$ 53. -3 is a solution

54. -1 is not a solution 55. 2 is not a solution 56. $\left\{\frac{55}{216}\right\}$

57. $\{66\}$ 58. $\left\{-\frac{37}{6}\right\}$ 59. $\left\{\frac{7}{20}\right\}$ 60. $\left\{-\frac{3}{29}\right\}$

61. $p = \frac{4m-6n+26}{3}$ 62. $C = \frac{5}{9}(F-32)$

63. $V_1 = \frac{P_2V_2T_1}{P_1T_2}$ 64. $R_2 = \frac{R_tR_1}{R_1-R_t}$ 65. 2 days

66. 60 mph, automobile; 90 mph, train 67. 3 mph 68. $-\frac{1}{28}$

Chapter 4 cumulative test

1. 25 2. -7 3. $\frac{4}{15}$ 4. 5 5. $\frac{13}{30}$ 6. $7x+11$

7. $-24a^6b^5$ 8. $x^3+2x^2-3x+20$ 9. 2 10. $\frac{x^2+6x-16}{x^2+3x}$

11. $\frac{-10y-13}{24}$ 12. $10x^2+39x-27$ 13. $16x^2-40x+25$

14. $9y^2-25$ 15. $2x^2+9x+27+\frac{80}{x-3}$

16. $\frac{6a^2+11a-10}{4a^2+23a-35}$ 17. $\frac{-2x+19}{(2x-3)(x+1)(2x+1)}$

18. $\left\{y|y \in R, y \neq \frac{3}{2}\right\}$ 19. $\left\{x|x \in R, x \neq \frac{1}{2},-\frac{1}{2}\right\}$

20. $\{x|x \in R, x \neq -5,5\}$ 21. $\{14\}$ 22. $\left\{-\frac{11}{24}\right\}$ 23. $\left\{-\frac{1}{8}\right\}$

24. $-\frac{13}{14}$ 25. $\frac{4x^2-3x-10}{5x^2+2x-16}$ 26. $\{x|x \leq -1\}$

27. $\left\{y|y < \frac{17}{2}\right\}$ 28. $\{z|z > -5\}$ 29. $\left\{x|x \geq \frac{97}{23}\right\}$

30. $-\frac{3a}{2b^2}$ 31. $\frac{p+4}{p+3}$ 32. $-\frac{3}{2}$ 33. $\left\{-\frac{1}{9}\right\}$ 34. $\frac{1}{2}$ or 6

35. $P(-2) = 27$

Chapter 5

Exercise 5-1

Answers to odd-numbered problems

1. 4.243 3. -5.745 5. $\sqrt[3]{9}$ 7. $\sqrt{x}$ 9. $\sqrt[5]{b^4}$ 11. 4

13. 16 15. 27 17. 64 19. $\frac{1}{2}$ 21. $\frac{1}{2}$ 23. $\frac{1}{9}$ 25. $\frac{-1}{8}$

27. $\frac{1}{\sqrt[4]{x^3}}$ 29. $a^{4/7}$ 31. $x^{1/5}$ 33. -8 35. $|-4| = 4$

37. $|2x-y|$ 39. 256 41. $E = \frac{T}{\sqrt{(x^2+r^2)^3}}$

43. 24 miles per hour

Solutions to trial exercise problems

13. $(-64)^{2/3} = (\sqrt[3]{-64})^2 = (-4)^2 = 16$

18. $(-27)^{1/3} = \dfrac{1}{(-27)^{1/3}} = \dfrac{1}{\sqrt[3]{-27}} = \dfrac{1}{-3} = -\dfrac{1}{3}$

Review exercises

1. a^7 2. x^{10} 3. $4a^6b^8$ 4. $\dfrac{1}{a^4}$ 5. 3 6. -9 7. $\dfrac{1}{a^7}$

8. $\dfrac{1}{x^2}$

Exercise 5–2

Answers to odd-numbered problems

1. 2 3. $b^{17/12}$ 5. 5 7. $a^{1/12}$ 9. $a^{8/15}$ 11. x^3 13. $\dfrac{1}{x}$

15. $b^{1/3}$ 17. $x^{1/4}$ 19. $8y^3$ 21. $a^2b^{2/3}$ 23. $8a^{15/2}b^{3/2}$

25. $\dfrac{1}{3a^4b}$ 27. $\dfrac{1}{y^{1/12}}$ 29. $\dfrac{1}{b^{1/4}}$ 31. x^2 33. $a^{2/3}$ 35. $a^{1/2}b^{1/4}$

37. $a^{1/2}b^{1/2}$ 39. $b^{11/6}$ 41. $\dfrac{1}{a^{1/3}b^{1/4}}$ 43. 13 in. 45. 30 mph

47. 18 miles 49. take the square root three times

Solutions to trial exercise problems

9. $(a^{2/3})^{4/5} = a^{2/3 \cdot 4/5} = a^{8/15}$ 13. $(x^{-1/4})^4 = x^{-1/4 \cdot 4} = x^{-1} = \dfrac{1}{x}$

19. $(16y^4)^{3/4} = (\sqrt[4]{16y^4})^3 = (2y)^3 = 8y^3$ 27. $\dfrac{y^{1/4}}{y^{1/3}} = y^{1/4 - 1/3}$
$= y^{3/12 - 4/12} = y^{-1/12} = \dfrac{1}{y^{1/12}}$ 41. $\dfrac{a^{-2/3}b^{1/2}}{a^{-1/3}b^{3/4}} = a^{-2/3 - (-1/3)}b^{1/2 - 3/4}$
$= a^{-1/3}b^{2/4 - 3/4} = a^{-1/3}b^{-1/4} = \dfrac{1}{a^{1/3}b^{1/4}}$

Review exercises

1. $32a^7$ 2. $16x^4y^3$ 3. $375x^3y^4$ 4. $54a^3b^5$ 5. $375a^6b^8$
6. $32a^6b^6$

Exercise 5–3

Answers to odd-numbered problems

1. $2\sqrt{5}$ 3. $2\sqrt[3]{3}$ 5. $a\sqrt[4]{a}$ 7. a^2 9. c 11. $5xy^4\sqrt{xy}$
13. $5a^3c^4\sqrt{2bc}$ 15. $3ac^4\sqrt[3]{b^2}$ 17. $3ab^3\sqrt[3]{3a^2b^2}$ 19. $2a^2c^2\sqrt[5]{b^4c^2}$
21. $x + 3$ 23. $3a + 1$ 25. $9\sqrt{2}$ 27. 7 29. 12 31. $2\sqrt[3]{9}$
33. $3a\sqrt{5}$ 35. $a\sqrt[3]{a}$ 37. x 39. $2x\sqrt[5]{x^2}$ 41. $5ab\sqrt[3]{3a}$
43. $5x^2y^3\sqrt[3]{3y}$ 45. $2xy\sqrt[4]{2}$ 47. $\sqrt{y}$ 49. $y\sqrt[4]{y^3}$ 51. $\sqrt{2y}$
53. $\sqrt[3]{2ab^2}$ 55. $4|x|$ 57. $7|bc|$ 59. $|a - 4|$ 61. $|a|\sqrt{b}$
63. $3b\sqrt[3]{a}$ 65. $3|a|\sqrt[4]{b^3}$ 67. $3\sqrt[3]{3}$ in. 69. 3 in. 71. 6 units
73. $5\sqrt{6}$ amperes 75. 5 m 77. 13 in. 79. $\sqrt{41}$ in.
81. $3\sqrt{13}$ ft 83. 20 mm 85. $2\sqrt{21}$ cm 87. 5 feet
89. 44.27 meters per second

Solutions to trial exercise problems

10. $\sqrt{9x^2y^5} = \sqrt{9x^2y^2y^2y} = \sqrt{9}\sqrt{x^2}\sqrt{y^2}\sqrt{y^2}\sqrt{y} = 3xyy\sqrt{y}$
$= 3xy^2\sqrt{y}$ 24. $\sqrt{x^2 + y^2}$. Will not simplify because x^2 and y^2 are terms, not factors. 40. $\sqrt[3]{4a^2b}\sqrt[3]{4a^2b^2} = \sqrt[3]{4a^2b \cdot 4a^2b^2}$
$= \sqrt[3]{16a^4b^3} = \sqrt[3]{8 \cdot 2 \cdot a^3 \cdot a \cdot b^3} = \sqrt[3]{8}\sqrt[3]{2}\sqrt[3]{a^3}\sqrt[3]{a}\sqrt[3]{b^3}$
$= 2 \cdot \sqrt[3]{2} \cdot a \cdot \sqrt[3]{a} \cdot b = 2ab\sqrt[3]{2a}$ 48. $\sqrt[6]{b^{10}} = b^{10/6} = b^{5/3}$
$= \sqrt[3]{b^5} = \sqrt[3]{b^3}\sqrt[3]{b^2} = b\sqrt[3]{b^2}$ 67. $h = \sqrt[3]{\dfrac{12I}{b}} = \sqrt[3]{\dfrac{12(27)}{(4)}}$
$= \sqrt[3]{3(27)} = \sqrt[3]{27}\sqrt[3]{3} = 3\sqrt[3]{3}$ in. 79. $c = \sqrt{a^2 + b^2}$
$= \sqrt{(5)^2 + (4)^2} = \sqrt{25 + 16} = \sqrt{41}$ in.

Review exercises

1. 9 2. 4 3. 2 4. a 5. x 6. a 7. 2 8. x

Exercise 5–4

Answers to odd-numbered problems

1. $\dfrac{4}{5}$ 3. $\dfrac{\sqrt{7}}{3}$ 5. $\dfrac{2}{3}$ 7. $\dfrac{a^3}{3}$ 9. $\dfrac{\sqrt[3]{2x}}{y^5}$ 11. $\dfrac{x^3}{yz^2}$

13. $\dfrac{2x}{y^2}$ 15. $2x^2y\sqrt[5]{2}$ 17. $\dfrac{\sqrt{2}}{2}$ 19. $\dfrac{3\sqrt{10}}{10}$ 21. $\dfrac{\sqrt{6}}{6}$

23. $\dfrac{3\sqrt{2}}{10}$ 25. $\sqrt{2}$ 27. $3\sqrt{2}$ 29. $\dfrac{3\sqrt[3]{2}}{2}$ 31. $\dfrac{2\sqrt[5]{3}}{3}$

33. $\dfrac{3\sqrt[4]{4}}{4} = \dfrac{3\sqrt{2}}{4}$ 35. $\dfrac{x\sqrt{y}}{y}$ 37. $\dfrac{\sqrt{c}}{c}$ 39. $\dfrac{a\sqrt[3]{b}}{b}$

41. $\dfrac{\sqrt[3]{ab}}{b}$ 43. $\dfrac{2x\sqrt[5]{y^3}}{y}$ 45. $a\sqrt[5]{b}$ 47. $\dfrac{\sqrt{2xyz}}{yz}$

49. $\dfrac{2\sqrt[3]{xyz^2}}{yz}$ 51. $\dfrac{\sqrt[7]{8x^5y^4}}{2xy}$ 53. $\dfrac{\sqrt[5]{x^3y}}{y}$ 55. $\sqrt[3]{x^2y}$

57. $b\sqrt[7]{b^3c^4}$ 59. $\dfrac{\sqrt{xy}}{2}$ 61. $\dfrac{2a^2}{b}$ 63. $\dfrac{y^2\sqrt[5]{z}}{xz}$ 65. 6 units

67. $\dfrac{2\sqrt{3gh}}{3}$ 69. $\dfrac{c\sqrt{2}}{2}$ 71. $\dfrac{2f\sqrt{3}}{3}$ 73. $\dfrac{2\sqrt{2\pi kmT}}{\pi m}$

Solutions to trial exercise problems

23. $\sqrt{\dfrac{9}{50}} = \dfrac{\sqrt{9}}{\sqrt{50}} = \dfrac{3}{\sqrt{25 \cdot 2}} = \dfrac{3}{5\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{3\sqrt{2}}{5 \cdot 2} = \dfrac{3\sqrt{2}}{10}$

44. $\dfrac{x^2}{\sqrt[3]{x^2}} \cdot \dfrac{\sqrt[3]{x}}{\sqrt[3]{x}} = \dfrac{x^2\sqrt[3]{x}}{x} = x\sqrt[3]{x}$

51. $\sqrt[7]{\dfrac{1}{16x^2y^3}} = \dfrac{\sqrt[7]{1}}{\sqrt[7]{16x^2y^3}} = \dfrac{1}{\sqrt[7]{2^4x^2y^3}} \cdot \dfrac{\sqrt[7]{2^3x^5y^4}}{\sqrt[7]{2^3x^5y^4}} = \dfrac{\sqrt[7]{8x^5y^4}}{2xy}$

59. $\sqrt{\dfrac{2y}{x}}\sqrt{\dfrac{x^2}{8}} = \sqrt{\dfrac{2y}{x} \cdot \dfrac{x^2}{8}} = \sqrt{\dfrac{xy}{4}} = \dfrac{\sqrt{xy}}{\sqrt{4}} = \dfrac{\sqrt{xy}}{2}$

Review exercises

1. $9x^2 + x$ 2. $2a^2b - ab^2$ 3. $5a^3 + 2a^2$ 4. $3x^2y - 4xy^2$

5. $\dfrac{13}{4}$ 6. $\dfrac{7}{2x}$ 7. $\dfrac{2}{3a}$ 8. $\dfrac{13}{6x}$

Exercise 5–5

Answers to odd-numbered problems

1. $11\sqrt{5}$ 3. $7\sqrt{3}$ 5. $8\sqrt{5}$ 7. $-\sqrt{10}$ 9. $7\sqrt[3]{4}$
11. $6\sqrt[4]{3}$ 13. $5\sqrt[5]{12} - \sqrt[5]{16}$ 15. $4\sqrt{3x} - 4\sqrt{2x}$
17. $-\sqrt{5}$ 19. $-3\sqrt{3}$ 21. $13\sqrt{3}$ 23. $\sqrt{3}$ 25. $5\sqrt[3]{2}$
27. $6\sqrt[3]{2} + 10\sqrt[3]{3}$ 29. $3\sqrt[3]{3} + 10\sqrt[3]{2}$ 31. $\sqrt{2x}$
33. $37a\sqrt{b}$ 35. $70a\sqrt{b} - 11\sqrt{2b}$ 37. $5\sqrt[4]{a}$
39. $-23\sqrt[3]{a^2}$ 41. $4a^2\sqrt[3]{b}$ 43. $2a^2\sqrt{ab}$ 45. $2a^2b^2\sqrt{ab}$

47. $\dfrac{1 + 2\sqrt{5}}{5}$ 49. $\dfrac{4 - 6\sqrt{3}}{9}$ 51. $\dfrac{4\sqrt{5} + 5\sqrt{6}}{10}$

53. $\dfrac{6\sqrt{5} + \sqrt{10}}{5}$ 55. $\dfrac{7\sqrt{3}}{12}$ 57. $\dfrac{\sqrt{x}}{2x}$

59. $\dfrac{5\sqrt{xy} - 4y\sqrt{x}}{xy}$ 61. 17 units 63. $13\sqrt{13} \approx 46.87$ feet

65. 18.23 feet

Solutions to trial exercise problems

12. $7\sqrt[3]{11} - 3\sqrt[3]{7} + 2\sqrt[3]{11} = (7\sqrt[3]{11} + 2\sqrt[3]{11}) - 3\sqrt[3]{7}$ $= 9\sqrt[3]{11} - 3\sqrt[3]{7}$ **16.** $\sqrt{12} + 4\sqrt{3} = \sqrt{4 \cdot 3} + 4\sqrt{3}$ $= 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$ **41.** $\sqrt[3]{a^6 b} + 3a^2\sqrt[3]{b}$ $= \sqrt[3]{a^3 a^3 b} + 3a^2\sqrt[3]{b} = a \cdot a\sqrt[3]{b} + 3a^2\sqrt[3]{b} = a^2\sqrt[3]{b} + 3a^2\sqrt[3]{b}$ $= 4a^2\sqrt[3]{b}$ **52.** $\frac{4}{\sqrt{7}} - \frac{2}{\sqrt{14}} = \frac{4}{\sqrt{7}}\frac{\sqrt{7}}{\sqrt{7}} - \frac{2}{\sqrt{14}}\frac{\sqrt{14}}{\sqrt{14}}$ $= \frac{4\sqrt{7}}{7} - \frac{2\sqrt{14}}{14} = \frac{4\sqrt{7}}{7} - \frac{\sqrt{14}}{7} = \frac{4\sqrt{7} - \sqrt{14}}{7}$

Review exercises

1. $6a^2 - 12a$ **2.** $6x^2 + xy - y^2$ **3.** $a^2 - 2ab + b^2$ **4.** $a^2 + 2ab + b^2$ **5.** $4a^2 + 4ab + b^2$ **6.** $x^2 - y^2$ **7.** $9a^2 - b^2$ **8.** $16x^2 - 9y^2$

Exercise 5-6

Answers to odd-numbered problems

1. $3\sqrt{5} + 3\sqrt{3}$ **3.** $12\sqrt{7} + 4\sqrt{2}$ **5.** $\sqrt{6} + \sqrt{15}$ **7.** $3\sqrt{6} - \sqrt{22}$ **9.** $35\sqrt{10} - 20\sqrt{15}$ **11.** $5\sqrt{3} - 10$ **13.** $14\sqrt{5} - 56\sqrt{2}$ **15.** $x + \sqrt{xy}$ **17.** $6x\sqrt{y} - 15x$ **19.** $5x\sqrt{y} + 20y\sqrt{x}$ **21.** $9 + 5\sqrt{3}$ **23.** $20 + 9\sqrt{x} + x$ **25.** $3 + 2\sqrt{y} - 8y$ **27.** 14 **29.** -3 **31.** -2 **33.** $a - b^2$ **35.** $9x - 16y$ **37.** $11 - 4\sqrt{7}$ **39.** $91 - 40\sqrt{3}$ **41.** $-4a + 4b\sqrt{a} + b^2$ **43.** $20x - \sqrt{xy} - y$ **45.** $2 - \sqrt{3}$ **47.** $\frac{12 - 3\sqrt{6}}{5}$ **49.** $\sqrt{10} + \sqrt{6}$ **51.** $\frac{6 + 3\sqrt{3}}{2}$ **53.** $\frac{-\sqrt{3} - 3\sqrt{2}}{5}$ **55.** $\frac{21\sqrt{2} + 4\sqrt{7}}{55}$ **57.** $\frac{x - \sqrt{xy}}{x - y}$ **59.** $\frac{a - \sqrt{a}}{a - 1}$ **61.** $\frac{x - 2y\sqrt{x} + y^2}{x - y^2}$ **63.** $\frac{\sqrt{ab} + \sqrt{a}}{b - 1}$ **65.** $\frac{2\sqrt{a} + 2\sqrt{ab}}{1 - b}$ **67.** $\frac{\sqrt{b} + 1}{b - 1}$ **69.** $\frac{T\sqrt{x^2 + r^2}}{(x^2 + r^2)^2}$

Solutions to trial exercise problems

10. $\sqrt{6}(\sqrt{2} + \sqrt{3}) = \sqrt{6 \cdot 2} + \sqrt{6 \cdot 3} = \sqrt{12} + \sqrt{18}$ $= \sqrt{4 \cdot 3} + \sqrt{9 \cdot 2} = 2\sqrt{3} + 3\sqrt{2}$ **27.** $(4 - \sqrt{2})(4 + \sqrt{2})$ $= 16 + 4\sqrt{2} - 4\sqrt{2} - \sqrt{2}\sqrt{2} = 16 - 2 = 14.$ Since these are conjugate factors, we could have written $(4 - \sqrt{2})(4 + \sqrt{2}) = (4)^2 - (\sqrt{2})^2 = 16 - 2 = 14.$

47. $\frac{6}{4 + \sqrt{6}} = \frac{6}{4 + \sqrt{6}} \cdot \frac{4 - \sqrt{6}}{4 - \sqrt{6}} = \frac{6(4 - \sqrt{6})}{(4)^2 - (\sqrt{6})^2}$ $= \frac{6(4 - \sqrt{6})}{16 - 6} = \frac{6(4 - \sqrt{6})}{10} = \frac{3(4 - \sqrt{6})}{5} = \frac{12 - 3\sqrt{6}}{5}$

53. $\frac{\sqrt{6}}{\sqrt{2} - 2\sqrt{3}} = \frac{\sqrt{6}}{\sqrt{2} - 2\sqrt{3}} \cdot \frac{\sqrt{2} + 2\sqrt{3}}{\sqrt{2} + 2\sqrt{3}}$ $= \frac{\sqrt{6}(\sqrt{2} + 2\sqrt{3})}{(\sqrt{2})^2 - (2\sqrt{3})^2} = \frac{\sqrt{6 \cdot 2} + 2\sqrt{3 \cdot 6}}{2 - 4 \cdot 3} = \frac{\sqrt{12} + 2\sqrt{18}}{2 - 12}$ $= \frac{\sqrt{4 \cdot 3} + 2\sqrt{9 \cdot 2}}{-10} = \frac{2\sqrt{3} + 2 \cdot 3\sqrt{2}}{-10} = \frac{2(\sqrt{3} + 3\sqrt{2})}{-10}$ $= \frac{-(\sqrt{3} + 3\sqrt{2})}{5} = \frac{-\sqrt{3} - 3\sqrt{2}}{5}$

60. $\frac{\sqrt{a} + b}{\sqrt{a} - b} = \frac{\sqrt{a} + b}{\sqrt{a} - b} \cdot \frac{\sqrt{a} + b}{\sqrt{a} + b} = \frac{(\sqrt{a} + b)(\sqrt{a} + b)}{(\sqrt{a})^2 - (b)^2}$ $= \frac{\sqrt{a}\sqrt{a} + b\sqrt{a} + b\sqrt{a} + b^2}{a - b^2} = \frac{a + 2b\sqrt{a} + b^2}{a - b^2}$

63. $\frac{a}{\sqrt{ab} - \sqrt{a}} = \frac{a}{\sqrt{ab} - \sqrt{a}} \cdot \frac{\sqrt{ab} + \sqrt{a}}{\sqrt{ab} + \sqrt{a}}$ $= \frac{a(\sqrt{ab} + \sqrt{a})}{(\sqrt{ab})^2 - (\sqrt{a})^2} = \frac{a(\sqrt{ab} + \sqrt{a})}{ab - a} = \frac{a(\sqrt{ab} + \sqrt{a})}{a(b - 1)}$ $= \frac{\sqrt{ab} + \sqrt{a}}{b - 1}$

Review exercises

1. $2a^2 - ab - b^2$ **2.** $a^2 - 3ab + 2b^2$ **3.** $4a^2 - 9b^2$ **4.** $a^2 + 6a + 9$ **5.** 4 **6.** $2\sqrt{5}$ **7.** 6 **8.** $6\sqrt{2}$

Exercise 5-7

Answers to odd-numbered problems

1. $3i$ **3.** $2i\sqrt{3}$ **5.** -9 **7.** -3 **9.** $-\sqrt{15}$ **11.** -2 **13.** -5 **15.** -6 **17.** $-3\sqrt{5}$ **19.** -1 **21.** 1 **23.** $-i$ **25.** $8 + 7i$ **27.** $-2 + 2i$ **29.** $1 + 8i$ **31.** i **33.** $5 - 2i$ **35.** $8 + 8i$ **37.** 13 **39.** $19 - 7i$ **41.** $19 + 17i$ **43.** 34 **45.** $7 - 24i$ **47.** $8i$ **49.** $5 - 4i$ **51.** $2 - i$ **53.** $\frac{7}{3} - \frac{5}{3}i$ **55.** $\frac{27}{26} - \frac{5}{26}i$ **57.** $\frac{4}{5} - \frac{3}{5}i$ **59.** $\frac{-4}{13} + \frac{19}{13}i$ **61.** $\frac{6}{5} - \frac{3}{5}i$ **63.** $\frac{4}{17} + \frac{18}{17}i$ **65.** $30 - 24i$ **67.** $0.571 - 0.143i$ **69.** $\frac{7}{5} + \frac{1}{5}i$ **71.** $\frac{60}{61} + \frac{50}{61}i$ **73.** $x \le 5$ **75.** $x < -11$

Solutions to trial exercise problems

5. $(3i)^2 = 3^2 i^2 = 9(-1) = -9$ **28.** $(4 - 5i) - (3 - 7i) = 4 - 5i - 3 + 7i = 1 + 2i$ **29.** $(2 + \sqrt{-49}) - (1 - \sqrt{-1}) = (2 + i\sqrt{49}) - (1 - i)$ $= (2 + 7i) - (1 - i) = 2 + 7i - 1 + i = 1 + 8i$ **32.** $[(2 + 5i) + (3 - 2i)] + (3 - i) = [2 + 5i + 3 - 2i]$ $+ (3 - i) = [5 + 3i] + (3 - i) = 5 + 3i + 3 - i = 8 + 2i$ **44.** $(2 + i)^2 = (2 + i)(2 + i) = 4 + 2i + 2i + i^2 = 4 + 4i$ $+ (-1) = 3 + 4i$ **48.** $\frac{3 - 2i}{i} = \frac{3 - 2i}{i}\frac{i}{i} = \frac{i(3 - 2i)}{i^2}$ $= \frac{3i - 2i^2}{-1} = \frac{3i - 2(-1)}{-1} = \frac{3i + 2}{-1} = -2 - 3i$

54. $\frac{4 - 3i}{1 + i} = \frac{4 - 3i}{1 + i} \cdot \frac{1 - i}{1 - i} = \frac{(4 - 3i)(1 - i)}{(1)^2 - (i)^2}$ $= \frac{4 - 4i - 3i + 3i^2}{1 - (-1)} = \frac{4 - 7i + 3(-1)}{2} = \frac{4 - 7i + (-3)}{2}$ $= \frac{1 - 7i}{2} = \frac{1}{2} - \frac{7}{2}i$

Review exercises

1. $(x - 6)^2$ **2.** $3x(x + 3)$ **3.** $(x + 4)(x - 4)$ **4.** $9(x + 2)(x - 2)$ **5.** $(x - 5)(x - 2)$ **6.** $(2x - 3)(x + 1)$ **7.** $(5x + 1)(x - 3)$ **8.** $(6x + 1)(x - 4)$

Chapter 5 review

1. 6 **2.** $\frac{1}{8}$ **3.** 9 **4.** $a^{11/12}$ **5.** $c^{1/4}$ **6.** $9x^2$ **7.** b^2
8. $a^{1/6}$ **9.** $4x^8y^4$ **10.** $x^{19/6}$ **11.** $a^{7/6}$ **12.** $2\sqrt{3}$
13. $5\sqrt{6}$ **14.** $x\sqrt[5]{x^2}$ **15.** $2ab\sqrt[3]{3b}$ **16.** $a\sqrt{a}$
17. $\sqrt[3]{2ab^2}$ **18.** 8 in. **19.** $\frac{7}{8}$ **20.** $\frac{4\sqrt{3}}{9}$ **21.** $\frac{2x\sqrt[3]{2y^2}}{z^2}$
22. $\frac{\sqrt{2}}{4}$ **23.** $3\sqrt{2}$ **24.** $\frac{2\sqrt[3]{5}}{5}$ **25.** $\frac{x\sqrt{y}}{y}$ **26.** $\frac{\sqrt[3]{a^2b^2}}{b}$
27. $\sqrt[5]{x^3}$ **28.** $\frac{\sqrt[3]{ab^2c}}{bc}$ **29.** $\frac{\sqrt[4]{a^3b^2}}{b}$ **30.** $\frac{x\sqrt{y}}{y}$ **31.** $8\sqrt{3}$
32. $8\sqrt{2}$ **33.** $29\sqrt{2a}$ **34.** $3x^2\sqrt{xy}$ **35.** $\frac{5\sqrt{6}-2\sqrt{3}}{6}$
36. $\frac{2\sqrt{ab}-b\sqrt{a}}{ab}$ **37.** $2\sqrt{3}-2\sqrt{5}$ **38.** $2a\sqrt{b}+4a$
39. $30-10\sqrt{5}$ **40.** 3 **41.** $4a-9b$ **42.** $9x+6y\sqrt{x}+y^2$
43. $\frac{\sqrt{6}-2}{2}$ **44.** $4-\sqrt{6}$ **45.** $\sqrt{2}+1$ **46.** $\frac{a^2b\sqrt{a}+ab\sqrt{ab}}{a^2-b}$
47. $7i$ **48.** $2i\sqrt{7}$ **49.** -4 **50.** -7 **51.** -6 **52.** -3
53. $-\sqrt{6}$ **54.** i **55.** $7+7i$ **56.** $-1-11i$ **57.** $18-i$
58. $-21+20i$ **59.** $4-3i$ **60.** $-2-\frac{7}{3}i$ **61.** $\frac{7}{5}-\frac{6}{5}i$
62. $\frac{69}{58}+\frac{13}{58}i$

Chapter 5 cumulative test

1. $(a-8)(a+1)$ **2.** $x(4x-3)$ **3.** $9(x-2)(x+2)$
4. $(2x+3)(x+4)$ **5.** $(3a+4)(a-5)$
6. $(3x+4)(2x+3)$ **7.** (a) 28, (b) -8 **8.** $\left\{\frac{2}{5}\right\}$
9. $\left\{x \mid x > -\frac{11}{3}\right\}$ **10.** $x=-6y$ **11.** $\left\{-1,\frac{5}{3}\right\}$
12. $\left\{x \mid x < -\frac{11}{2} \text{ or } x > \frac{5}{2}\right\}$ **13.** $\left\{-\frac{13}{5}\right\}$
14. $\left\{x \mid -\frac{3}{2} \le x \le 2\right\}$ **15.** $2a^2b\sqrt[3]{2b^2}$ **16.** $10-5i$
17. $4\sqrt{3}$ **18.** $a^{7/12}$ **19.** $2ab^2\sqrt[3]{a}$ **20.** $8a^9b^{12}c^3$ **21.** $3i\sqrt{2}$
22. $\sqrt{10}-\sqrt{6}$ **23.** $-\frac{7}{13}-\frac{9}{13}i$ **24.** $2a^3$ **25.** $\frac{x^3y^2}{3}$
26. $\frac{\sqrt[3]{2a^2b^2c}}{2bc}$ **27.** 7 inches **28.** 400 kg of 80% copper, 600 kg of 50% copper **29.** 375 meters per second
30. 1,166.4 meters per second

Chapter 6

Exercise 6-1

Answers to odd-numbered problems

1. $\{3,-4\}$ **3.** $\left\{\frac{1}{3},-\frac{5}{2}\right\}$ **5.** $\{2,3\}$ **7.** $\{5\}$ **9.** $\{-3,8\}$
11. $\{0,1\}$ **13.** $\{-3,3\}$ **15.** $\left\{-\frac{1}{2},2\right\}$ **17.** $\left\{-2,\frac{3}{4}\right\}$
19. $\{2\}$ **21.** $\{-8,1\}$ **23.** $\{-5,1\}$ **25.** $\left\{-\frac{1}{3},2\right\}$
27. $\left\{-\frac{5}{2},3\right\}$ **29.** $\{-6,1\}$ **31.** $\{-11,11\}$ **33.** $\{-7,7\}$
35. $\{-4\sqrt{2},4\sqrt{2}\}$ **37.** $\{-6\sqrt{2},6\sqrt{2}\}$ **39.** $\{-2\sqrt{2},2\sqrt{2}\}$
41. $\{-5\sqrt{2},5\sqrt{2}\}$ **43.** $\{-1,-13\}$ **45.** $\{12+11i,12-11i\}$
47. $\{-10+4\sqrt{3},-10-4\sqrt{3}\}$ **49.** $\left\{2,-\frac{5}{2}\right\}$
51. $\left\{\frac{3+2i\sqrt{21}}{10},\frac{3-2i\sqrt{21}}{10}\right\}$ **53.** $\{-8-b,-8+b\}$
55. $x=-2b,12b$ **57.** $x=-\frac{7a}{4},2a$ **59.** $x=y$
61. $x=-\frac{4y}{3},\frac{y}{2}$ **63.** (a) $t=4$ sec, (b) $t=2$ sec
65. $t=1$ sec **67.** $n=7$ **69.** 7 meters **71.** 8,15,17
73. 4,6; $-6,-4$ **75.** 7,9; $-7,-9$

Solutions to trial exercise problems

13. $-3y^2+27=0$
$-3(y^2-9)=0$
$-3(y+3)(y-3)=0$
$y=-3$ when $y+3=0$, $y=3$ when $y-3=0$
The solution set is $\{-3,3\}$.

21. $\frac{x}{2}+\frac{7}{2}=\frac{4}{x}$
Multiply each member by the LCM, $2x$.
$2x\cdot\frac{x}{2}+2x\cdot\frac{7}{2}=2x\cdot\frac{4}{x}$
$x^2+7x=8$
$x^2+7x-8=0$
$(x+8)(x-1)=0$
$x=-8$ when $x+8=0$ and
$x=1$ when $x-1=0$
The solution set is $\{-8,1\}$.

23. $(y+6)(y-2)=-7$
$y^2+4y-12=-7$
$y^2+4y-5=0$
$(y+5)(y-1)=0$
$y=-5$ when $y+5=0$
and $y=1$ when $y-1=0$
The solution set is $\{-5,1\}$

44. $(x-9)^2=-144$
$x-9=\sqrt{-144}=12i$ or $x-9=-\sqrt{-144}=-12i$
Then $x=9+12i$ or $x=9-12i$
The solution set is $\{9+12i, 9-12i\}$

52. $(x-7)^2=a^2, a>0$
$x-7=\sqrt{a^2}=a$ or $x-7=-\sqrt{a^2}=-a$
Then $x=7+a$ or $x=7-a$
$\{7-a,7+a\}$

56. $3x^2-13xy+4y^2=0$
$(3x-y)(x-4y)=0$
$x=\frac{y}{3}$ when $3x-y=0$ and
$x=4y$ when $x-4y=0$, so
$x=\frac{y}{3}$ or $x=4y$.

62. a. $P=100I-5I^2$
$420=100I-5I^2$
$5I^2-100I+420=0$
$5(I^2-20I+84)=0$
$5(I-6)(I-14)=0$
$I=6$ when $I-6=0$ and $I=14$ when $I-14=0$
So $P=420$ when $I=6$ amperes or $I=14$ amperes.

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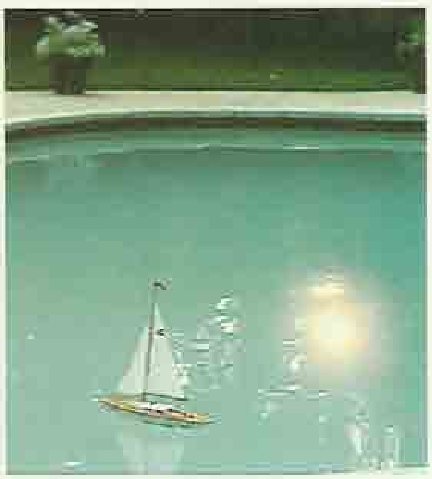

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